# Convolutional Neural Networks (CNN)

David I. Inouye

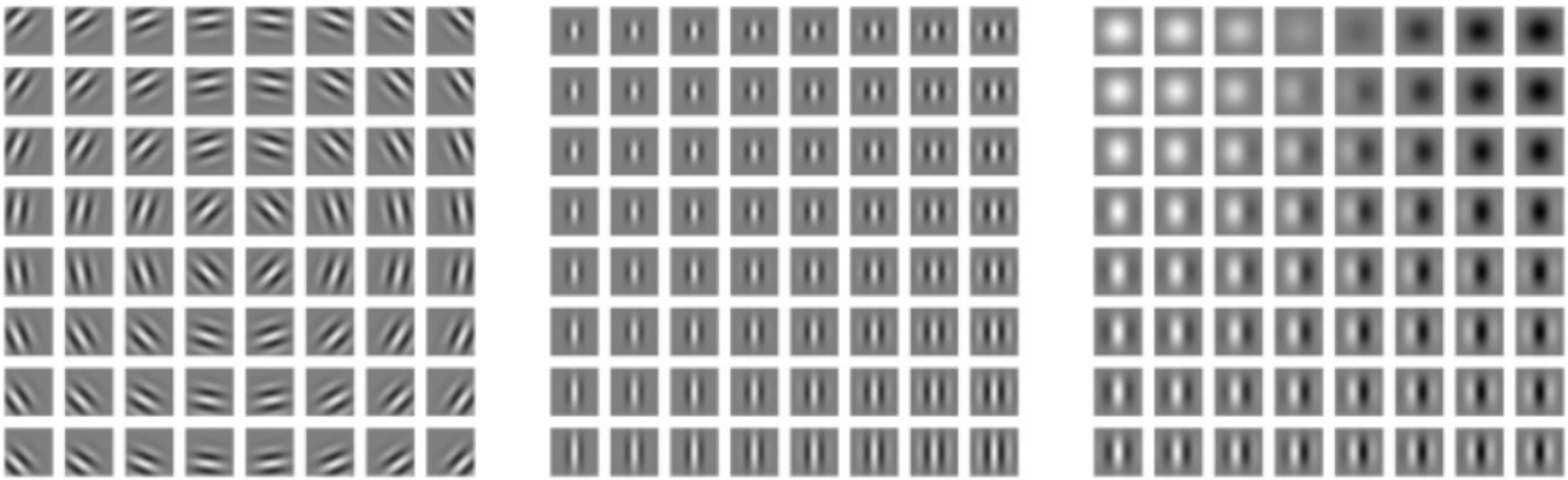
Thursday, February 9, 2023

# Why convolutional networks?

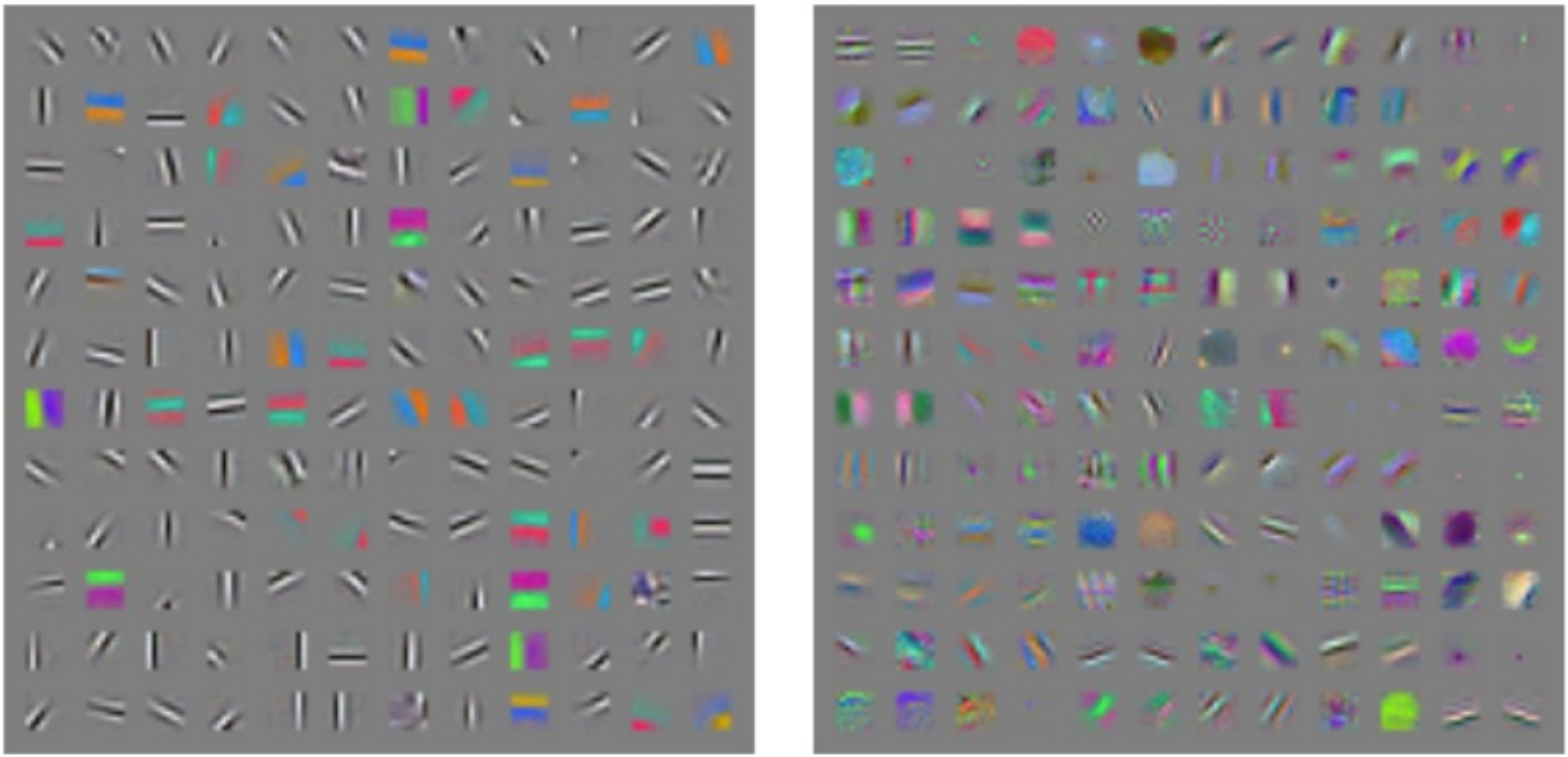
- Neuroscientific inspiration

- Computational reasons
  - Sparse computation (compared to full deep networks)
  - Shared parameters (only a small number of shared parameters)
  - Translation invariance

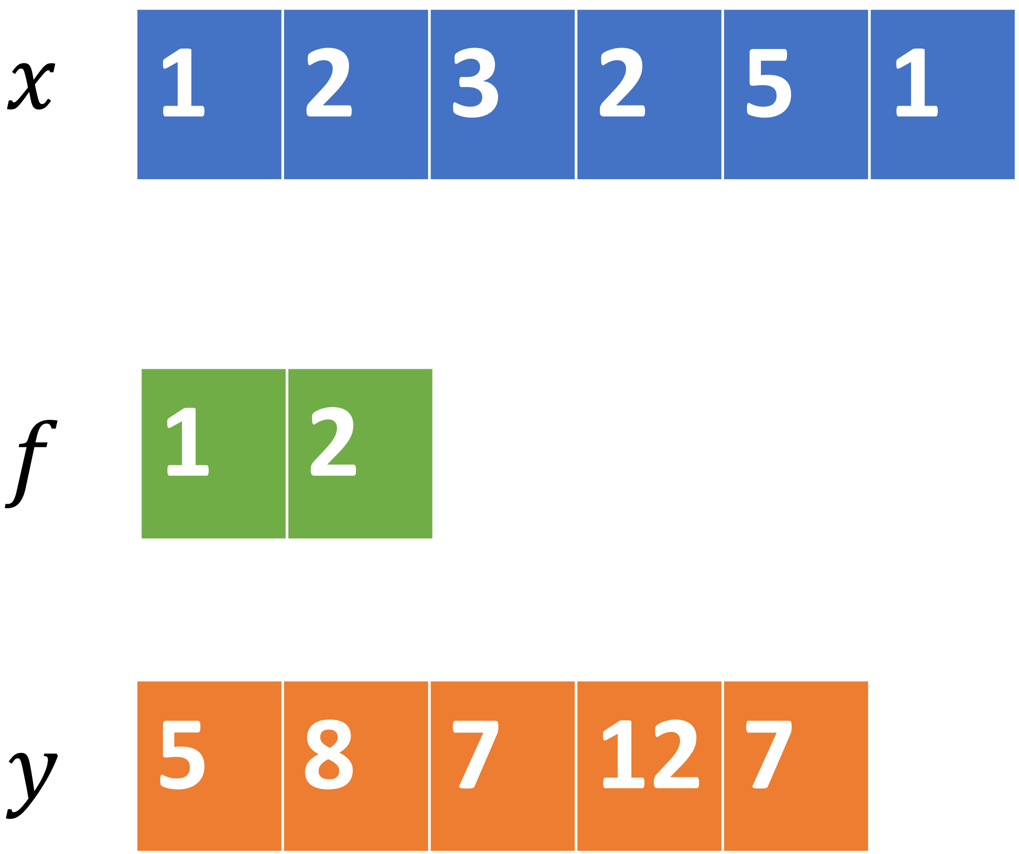
# Motivation for convolution networks: Gabor functions derived from neuroscience experiments are simple convolutional filters [DL, ch. 9]



# Convolutional networks automatically learn filters similar to Gabor functions [DL, ch. 9]



# 1D convolutions are similar but slightly different than signal processing / math convolutions



# Padding or stride parameters alter the computation and output shape

| $x$ | | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 2 | 5 | 1 |

| $f$ | | |
|---|---|---|
| | 1 | 2 |

Stride of 2

| $y$ | | | |
|---|---|---|---|
| | 5 | 7 | 7 |

# 1D convolutions are similar but slightly different than signal processing / math convolutions

$x$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 2 | 5 | 1 |

$f$

| | |
|---|---|
| 1 | 2 |

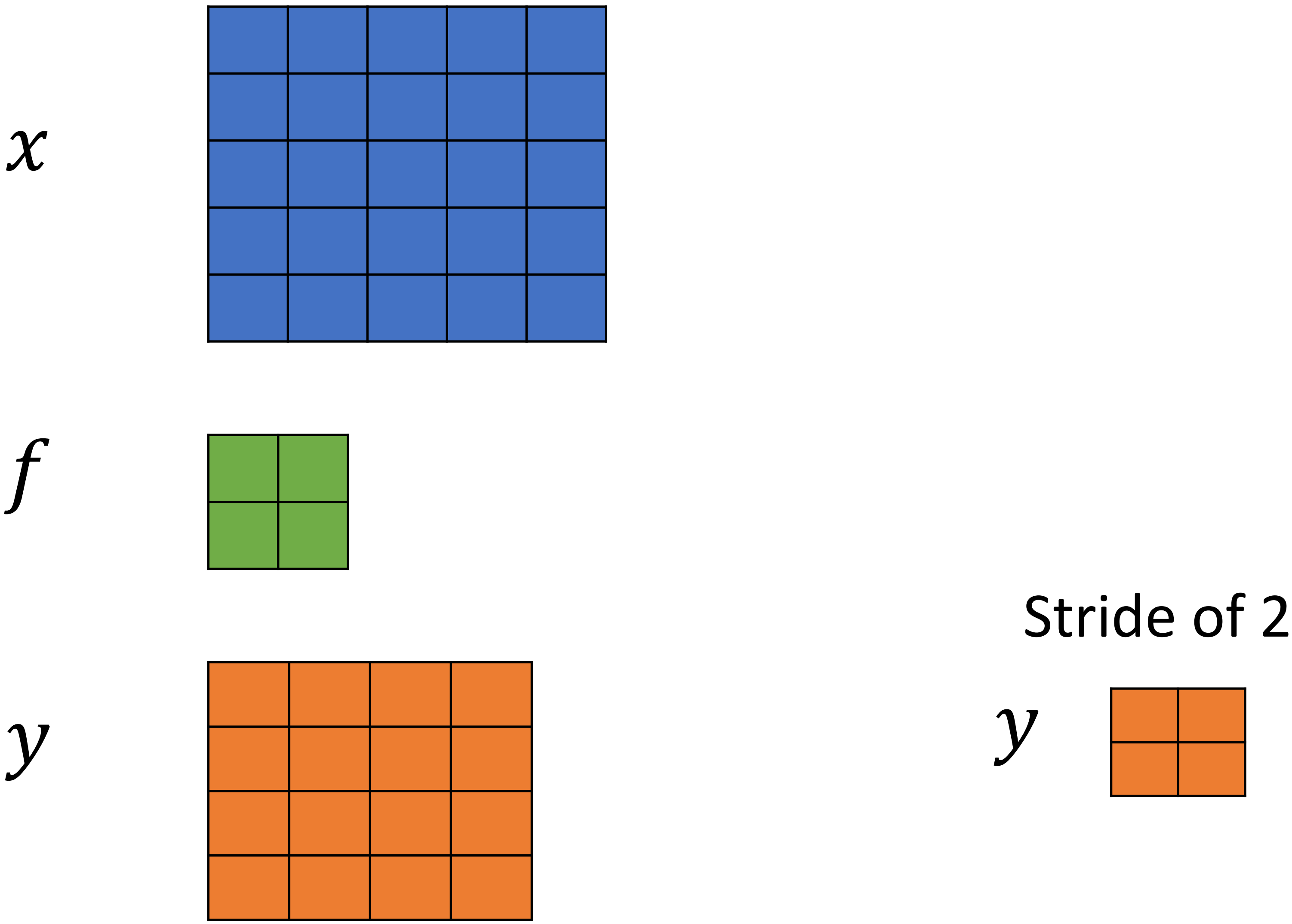
Zero padding of 1

$y$

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | 5 | 8 | 7 | 12 | 7 | 1 |

# Switch to demo of 1D

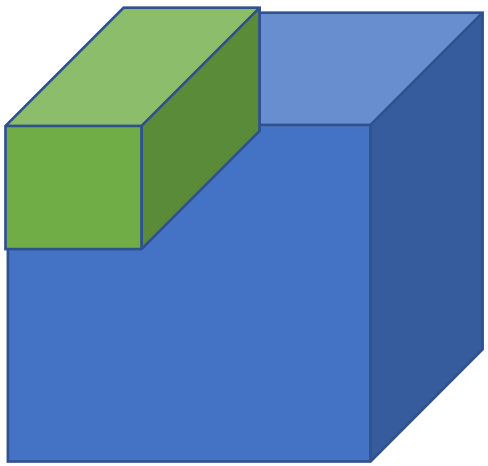
# 2D convolutions are simple generalizations to matrices



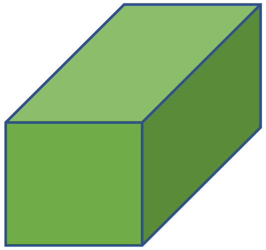
# Switch to demo of 2D

# 3D convolutions are similar but usually channel dimension is assumed

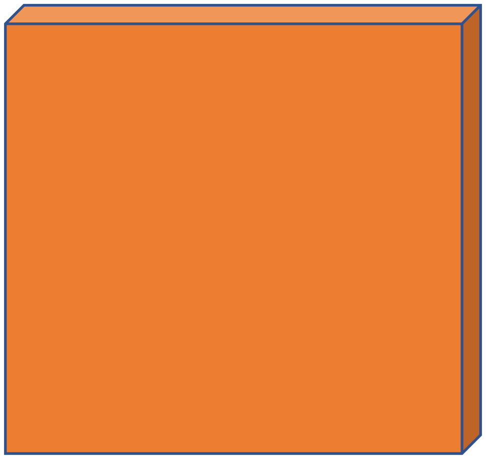
$$x \in \mathcal{R}^{c \times h \times w}$$



$$f \in \mathcal{R}^{c \times f_h \times f_w}$$



$$y \in \mathcal{R}^{1 \times h' \times w'}$$



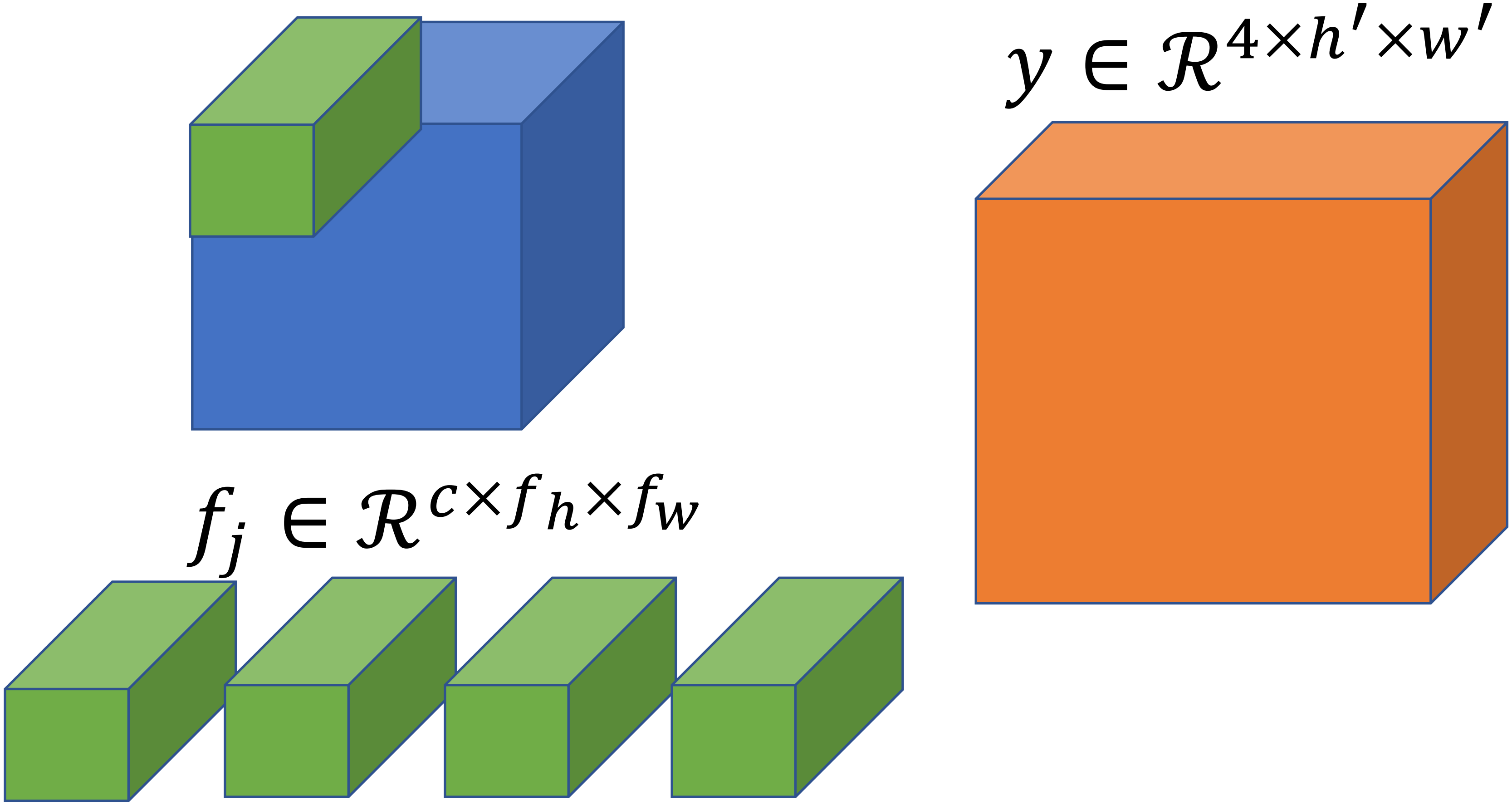
“$f_h \times f_w$ convolution” (channel dimension is assumed)

# Multiple convolutions increase the output channel dimension

$x \in \mathcal{R}^{c \times h \times w}$

$y \in \mathcal{R}^{4 \times h' \times w'}$

$f_j \in \mathcal{R}^{c \times f_h \times f_w}$



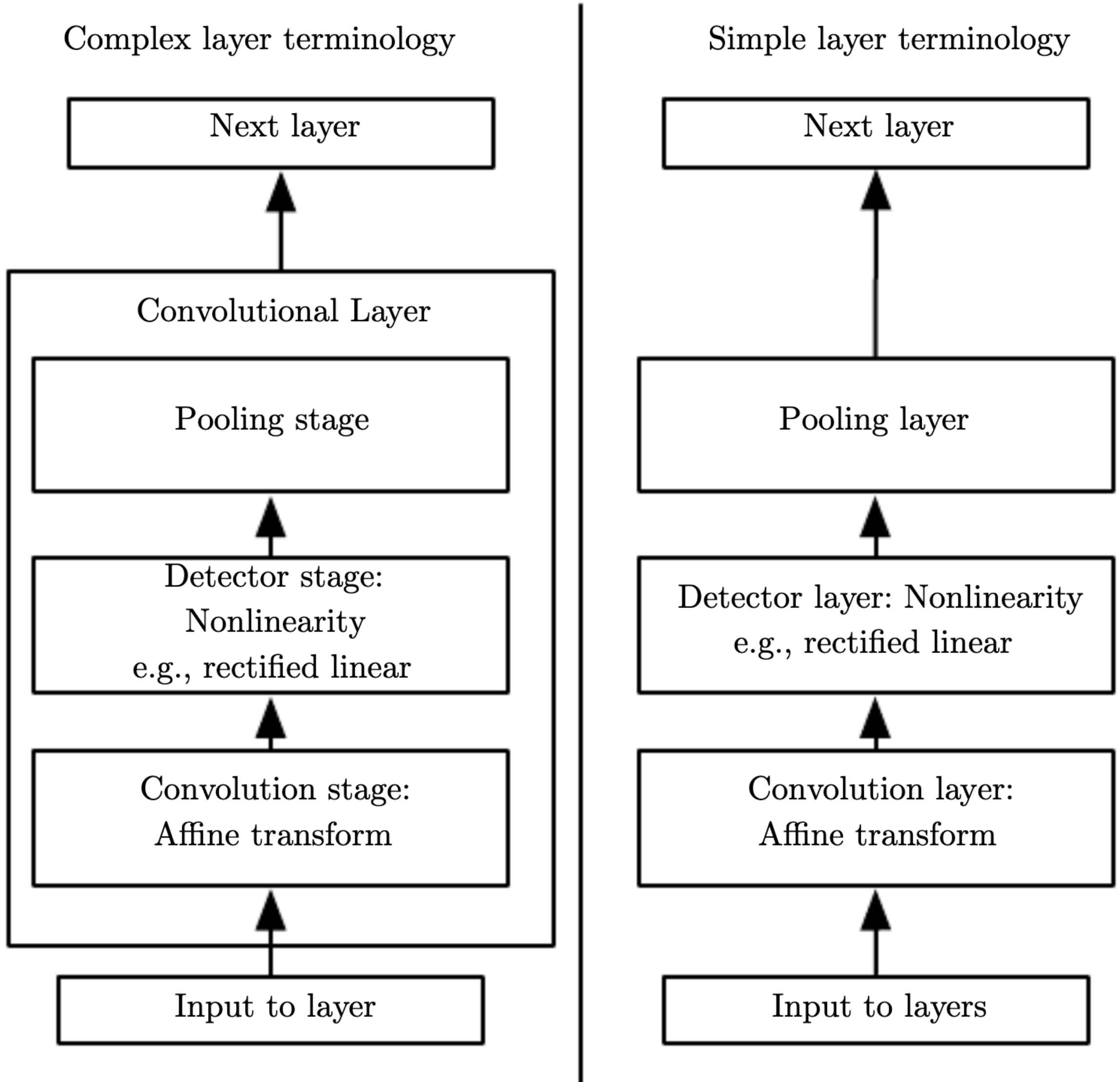
# Common convolution configurations

- Output has same height and width as input
  - 1 x 1 convolution with padding=0, stride=1
  - 3 x 3 convolution with padding=1, stride=1
  - 5 x 5 convolution with padding=2, stride=1

- Output has half the height and width of input
  - 2 x 2 convolution with padding=0, stride=2
  - 4 x 4 convolution with padding=1, stride=2

# Switch to demo of 3D, activation functions, and pooling

# Standard Convolutional Layer Terminology [DL, ch. 9]



# Demo of CIFAR-10 CNN in Pytorch

# Two important modern CNN architecture concepts: **batch normalization** and **residual networks**

# Batch normalization dynamically normalizes each feature to have zero mean and unit variance

- Basic idea: Normalize input batch of each layer **during the forward pass**

1. Input is **minibatch** of data $X^t \in \mathbb{R}^{m \times d}$ at iteration $t$
2. Compute mean and standard deviation for every feature

$$\mu_j^t = \mathbb{E}\left[x_j^t\right], \sigma_j^t = \sqrt{\mathbb{E}\left[\left(x_j^t - \mu_j^t\right)^2\right]}, \qquad \forall j \in \{1, \cdots, d\}$$

3. Normalize each feature (note **different for every batch**)

$$\tilde{x}_{i,j}^t = \frac{\left(x_{i,j}^t - \mu_j^t\right)}{\sigma_j^t}$$

4. Output $\tilde{X}^t$

Santurkar, S., Tsipras, D., Ilyas, A., & Madry, A. (2018). How does batch normalization help optimization?. In *Advances in Neural Information Processing Systems* (pp. 2483-2493).

# Because BatchNorm removes linear effects, extra linear parameters are also learned

- The form of this final update is:

$$\tilde{x}_{i,j}^t = \frac{\left(x_{i,j}^t - \mu_j^t\right)}{\sigma_j^t} \cdot \gamma_j + \beta_j$$

  - Where $\gamma_j$ and $\beta_j$ are learnable parameters
  - While $\mu_j^t$ and $\sigma_j^t$ are computed from the **minibatch**
- But how do we compute $\mu_j^t$ and $\sigma_j^t$ about during test time (i.e., no minibatch)?
- Use running average of mean and variance

$$\mu_{run}^t = \lambda \mu_{run}^{t-1} + (1-\lambda)\mu_{batch}^t$$
$${\sigma^2}_{run}^t = \lambda {\sigma^2}_{run}^{t-1} + (1-\lambda){\sigma^2}_{batch}^t$$

# For CNNs, the channel dimension is treated as a "feature"

- If the input minibatch tensor is $X^t \in \mathbb{R}^{m\times c\times h\times w}$, then the channel dimension $c$ is treated as a feature:

$$\mu_j^t = \mathbb{E}[x_j^t], \sigma_j^t = \sqrt{\mathbb{E}\left[\left(x_j^t - \mu_j^t\right)^2\right]},$$
$$\forall j \in \{1, \cdots, c\}$$

  - Where the mean is taken over **both** the batch dimension $m$ **and** the spatial dimensions $h$ and $w$
  - Called "Spatial Batch Normalization"
- Variants: Instance, Group or Layer Normalization

https://pytorch.org/docs/stable/nn.html#normalization-layers

# BatchNorm can stabilize and accelerate training of deep models

- To use in practice:
  - Only normalize batches during training (`model.train()`)
  - **Turn off** after training (`model.eval()`)
    - Uses running average of mean and variance
- Surprisingly effective at stabilizing training, reducing training time, and producing better models
- Not fully understood why it works

Santurkar, S., Tsipras, D., Ilyas, A., & Madry, A. (2018). How does batch normalization help optimization?. In *Advances in Neural Information Processing Systems* (pp. 2483-2493).

# Demo of batch normalization in PyTorch

# Residual networks add the input to the output of the CNN

- Most deep model layers have the form:
$$y = f(x)$$
  - Where $f$ could be any function including a convolutional layer like $f(x) = \sigma\left(\mathrm{Conv}\left(\sigma(\mathrm{Conv}(x))\right)\right)$
- Residual layers add back in the input
$$y = f(x) + x$$
  - Notice that $f(x)$ models the difference between $x$ and $y$ (hence the name **residual**)

He, K., Zhang, X., Ren, S., & Sun, J. (2016). Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition* (pp. 770-778).

# A residual network enables deeper networks because gradient information can flow between layers

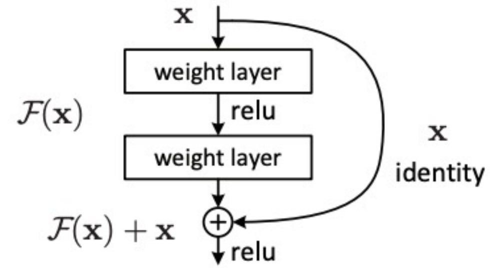
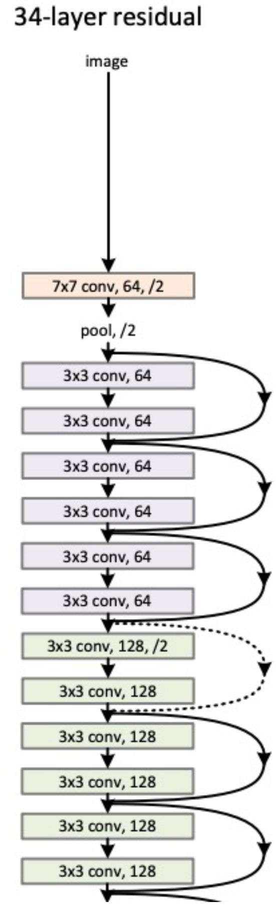



Figure 2. Residual learning: a building block.

- A data flow diagram shows the "shortcut" connections
- Consider composing 2 residual layers:
  - $z^{(1)} = f_1(x) + x$
  - $z^{(2)} = f_2\left(z^{(1)}\right) + z^{(1)}$
  - Or, equivalently $z^{(2)} = f_2(f_1(x) + x) + f_1(x) + x$
- If the residuals = 0, then this is merely the identity function




Images from: He, K., Zhang, X., Ren, S., & Sun, J. (2016). Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition* (pp. 770-778).

# Detail: If the dimensionality is not the same, then use either fully connected layer or convolution layer to match

- In the 1D case, suppose $f(x)\colon \mathbb{R}^d \to \mathbb{R}^m$, then we need to multiply $x$ by linear operator to match the dimension
$$y = f(x) + Wx, \qquad \text{where } W \in \mathbb{R}^{m\times d}$$

- Similarly, for images, if $f(x)\colon \mathbb{R}^{c\times h\times w} \to \mathbb{R}^{c'\times h'\times w'}$, we can apply a convolution layer to match the dimensions
$$y = f(x) + \operatorname{conv}(x),$$
$$\text{where } \operatorname{conv}(\cdot)\colon \mathbb{R}^{c\times h\times w} \to \mathbb{R}^{c'\times h'\times w'}$$

# Demo of CNN with very simple residual network

# U-Nets have an autoencoder structure with skip connections for **semantic segmentation** task

- Concatenation + convolution rather than residual skip connections
- ***Any*** (pretrained) classification backbone can be used for encoder
- State-of-the-art semantic segmentation are based on this idea

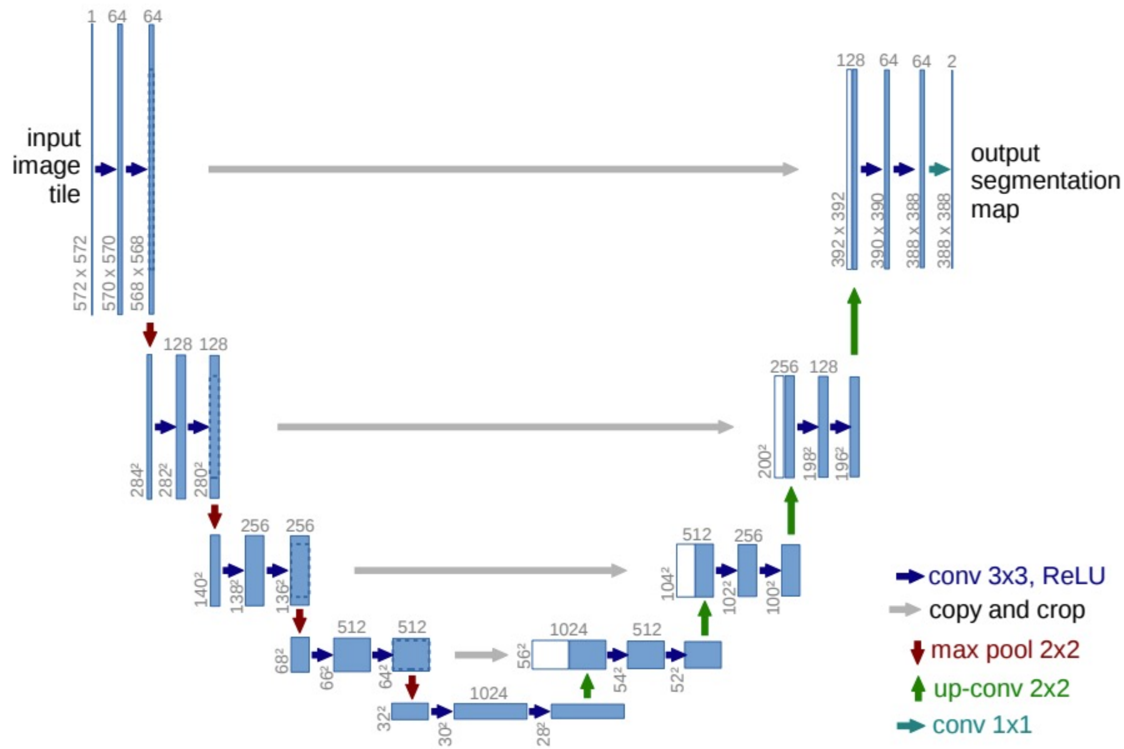



**Fig. 1.** U-net architecture (example for 32x32 pixels in the lowest resolution). Each blue box corresponds to a multi-channel feature map. The number of channels is denoted on top of the box. The x-y-size is provided at the lower left edge of the box. White boxes represent copied feature maps. The arrows denote the different operations.

Figure from: Ronneberger, O., Fischer, P., & Brox, T. (2015, October). U-net: Convolutional networks for biomedical image segmentation. In *International Conference on Medical image computing and computer-assisted intervention* (pp. 234-241). Springer, Cham.