Word Embeddings (Word2Vec)

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Fundamental question: How should words be represented so that relationships are encoded?

- Let $w_i$ denote the representation of the $i$-th word
- Synonyms should be similar
  - “Car” and “auto”, e.g., $\text{sim}(w_{\text{car}}, w_{\text{automobile}}) = 1$
- Antonyms should be dissimilar
  - “Good” and “bad”, e.g., $\text{sim}(w_{\text{good}}, w_{\text{bad}}) = -1$
- Related words would have some similarity
  - “School” and “book”, e.g., $\text{sim}(w_{\text{school}}, w_{\text{book}}) = 0.5$
Fundamental question: How should **words** be represented so that relationships are encoded?

- Encode more complex relationships like analogies
  - King is to man as queen is to ____.

https://samyzaf.com/ML/nlp/nlp.html
**Word embeddings** can be used to represent words with dense vectors that encode semantic relationships.

- Would one-hot vectors work?
  - i.e., $w_1 = [1,0,0,\ldots], w_2 = [0,1,0,\ldots], w_3 = [0,0,1,\ldots],$

- Goal: Automatically learn **dense** word embeddings that encode **semantic information** just given a bunch of text data (easy to obtain).

- Motivation: The **distributional hypothesis** in linguistics assumes words that occur in the same context are related.
Word embeddings are estimated via self-supervised learning—a type of unsupervised learning

- **Self-supervised learning** attempts to predict part of the data given other parts
  - Predict half of an image given the other half
  - Predict future from past
  - Predict past from future
  - Our case: Predict missing word(s) in sentence
- These are “pseudo” tasks
  - After training we don’t care about the model’s predictions
  - However, latent semantic structure emerges (i.e., the word embeddings are meaningful and useful)
Word2vec Task 1: Predict middle word given words before and after target word

- Sliding window across text
  - Joe fixed the fence while working in the yard.
  - Joe fixed _____ fence while working in the yard.
  - Joe fixed the _____ while working in the yard.
  - Joe fixed the fence _____ working in the yard.
  - Joe fixed the fence while _____ in the yard.

- The **target word** is the word to be predicted
- The **context** is the words before and after
Word2vec Task 1: **Continuous Bag-of-Words (CBOW)** architecture is a simple linear model

- CBOW adds the word embeddings of the context together (i.e., BoW) and then tries to predict

\[
\log p(y|C) = \log \sigma (A (\sum_{i \in C} w_i) + b)
\]

- The CBOW can be seen to be a

\[
\log p(y|C) = \log \sigma (A (W C) 1_{2m} + b)
\]

\[
A \in \mathbb{R}^{d \times k}, \quad W \in \mathbb{R}^{k \times d}, \quad C \in \{0,1\}^{d \times 2m}, \quad 1_{2m} = [1,1,1,\ldots]^T
\]

- Let $d, k, m$ denote the vocabulary size, the embedding dimension and the context size
Word2vec Task 2: Predict context given target word (mirror of task 1)

Could create paired training dataset

Example from http://jalammar.github.io/illustrated-word2vec/
Word2vec Task 2: However, this task is often too computationally expensive

- Predicting one word in a large vocabulary (think millions of words) is too expensive so simplify task to predict yes or no

<table>
<thead>
<tr>
<th>input word</th>
<th>target word</th>
</tr>
</thead>
<tbody>
<tr>
<td>not</td>
<td>thou</td>
</tr>
<tr>
<td>not</td>
<td>shalt</td>
</tr>
<tr>
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<td>make</td>
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<td>not</td>
<td>a</td>
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<tr>
<td>make</td>
<td>machine</td>
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</table>

<table>
<thead>
<tr>
<th>input word</th>
<th>output word</th>
<th>target</th>
</tr>
</thead>
<tbody>
<tr>
<td>not</td>
<td>thou</td>
<td>1</td>
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<td>not</td>
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</table>

Example from http://jalammar.github.io/illustrated-word2vec/
Word2vec Task 2: **Negative sampling** of words that should not be in context is required

- **Add random negative examples**

<table>
<thead>
<tr>
<th>input word</th>
<th>output word</th>
<th>target</th>
</tr>
</thead>
<tbody>
<tr>
<td>not</td>
<td>thou</td>
<td>1</td>
</tr>
<tr>
<td>not</td>
<td>aaron</td>
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<tr>
<td>not</td>
<td>taco</td>
<td>0</td>
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<td>not</td>
<td>shalt</td>
<td>1</td>
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<tr>
<td>not</td>
<td>make</td>
<td>1</td>
</tr>
</tbody>
</table>

Pick randomly from vocabulary (random sampling)

<table>
<thead>
<tr>
<th>Word</th>
<th>Count</th>
<th>Probability</th>
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</thead>
<tbody>
<tr>
<td>aardvark</td>
<td></td>
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</tr>
<tr>
<td>aarhus</td>
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<tr>
<td>aaron</td>
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<tr>
<td>taco</td>
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</tr>
<tr>
<td>thou</td>
<td></td>
<td></td>
</tr>
<tr>
<td>zyzzyva</td>
<td></td>
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</tbody>
</table>
Word2vec Task 2: The skip-gram model with negative sampling is simple

- The model merely trains a logistic regression

\[
\sum_{w_c \in \mathcal{C}} \left( \log p(w_c | w_{\text{mid}}) + \sum_{w_n \sim \text{Neg}} \log(1 - p(w_n | w_{\text{mid}})) \right)
\]

- This can be written as:

\[
\sum_{w_c \in \mathcal{C}} \left( \log \sigma(w_c^T w_{\text{mid}}) + \sum_{w_n \sim \text{Neg}} \log \left( 1 - \sigma(w_n^T w_{\text{mid}}) \right) \right)
\]

- where \( x_i \) is the one-hot encoding of the \( i \)-th word
- \( w_i' = C x_i \) is the output encoding
- \( w_i = W x_i \) is the input encoding
LSI and LDA topic models can be seen to produce a representation of words (decomposition approach)

Let’s examine the topic matrix again

“Automotive” topic may have high values on columns for “car”, “automobile” and “truck”.

Each word can show up in different topics. “book” may show up in “literature” and “school” topics/rows.
More recent word embeddings

- GloVe - https://nlp.stanford.edu/projects/glove/
Additional resources for word embeddings

- Nice tutorial on word2vec: http://jalammar.github.io/illustrated-word2vec/

- TensorFlow tutorial on Word2vec: https://www.tensorflow.org/tutorials/text/word2vec#next_steps
