

Quiz 0 (prerequisite quiz, ungraded)

Student Name

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Q1 Instructions

0 Points

This prerequisite quiz will NOT be taken for a grade. This is just to help you know if you are ready for this class or should consider taking it at a later time. I give a few general guidelines below though each person is different.

If you are comfortable solving 90% or more of these questions, then will probably do fine in this class.

If you are comfortable with about 80% of these questions, you will probably need to learn several background concepts independently on your own as you go through the course. The class is geared towards advanced students and independence in learning new concepts and skills is expected.

If you are comfortable with less than 70% of these questions, you may want to consider taking the class at a different time.

Note: Regular quizzes will not be this long. When giving numbers as short answers, please give in standard decimal notation with preceding 0 if needed but no trailing 0s (e.g., "0.15", "2.9", "100" but NOT "0.15000" NOR ".15" NOR "6.0").

Save Answer

Q2 Probability and random variables

6 Points

Q2.1

1 Point

Suppose X is a discrete random variable with the following probability mass function (PMF): $P(X = 1) = 0.2$, $P(X = 2) = 0.5$, $P(X = 3) = 0.3$.

What is the mean of this random variable (i.e., $E[X]$, where E denotes expectation)?

Enter your answer here

Save Answer

Q2.2
1 Point

For the same random variable X above, what is $E[X^2 + X - 1]$?

Enter your answer here

Save Answer

Q2.3
1 Point

Let the random variables X and Y have the following joint probability distribution:

$$\begin{aligned} P(X = 0, Y = 0) &= 0.1 & P(X = 1, Y = 0) &= 0.3 \\ P(X = 0, Y = 1) &= 0.4 & P(X = 1, Y = 1) &= 0.2 \end{aligned}$$

What is the marginal probability of $X = 0$ (i.e., what is $P(X = 0)$)?

Enter your answer here

Save Answer

Q2.4
1 Point

For the same random variables X and Y above.

What is the conditional probability of $Y = 1$ given that we know that $X = 1$ (i.e., what is $P(Y = 1|X = 1)$)?

Enter your answer here

Save Answer

Q2.5

1 Point

Suppose X is a continuous random variable that has the following probability density function (PDF):

$$f(x) = \begin{cases} 1.5, & \text{if } 0 \leq x < 0.5 \\ 0.5, & \text{if } 0.5 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

What is the cumulative distribution function (CDF) at $X = 0.5$ (i.e., $F(0.5)$ where F is the CDF of X)? (Remember that a CDF is defined as $F(x) = \int_{-\infty}^x f(z) dz$.)

Enter your answer here

Save Answer

Q2.6

1 Point

(Bonus, harder) Suppose the random variable U is distributed as a uniform distribution between 0 and 1, and let F_X and F_X^{-1} be the CDF and inverse CDF (or quantile) functions of an exponential random variable X with $\lambda = 2$ (see https://en.wikipedia.org/wiki/Exponential_distribution).

What is the PDF of the random variable Z defined as $Z \triangleq F_X^{-1}(U)$?

Enter your answer here

Save Answer

Q3 Linear algebra

7 Points

Q3.1
1 Point

Write the following linear equation in terms of the column vectors $\beta = [\beta_1, \beta_2, \beta_3]^T$ and $\mathbf{x} = [x_1, x_2, x_3]^T$, and the scalar y :

$$y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 .$$

Enter your answer here

Save Answer

Q3.2
1 Point

Suppose you have the following quadratic equation:

$$y = a_{11}x_1^2 + a_{12}x_1x_2 + a_{21}x_2x_1 + a_{22}x_2^2$$

Write this equation in terms of the matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, the column vector $\mathbf{x} = [x_1, x_2]^T$, and the scalar y :

Enter your answer here

Save Answer

Q3.3
1 Point

Let $\mathbf{x} = [1, 2, 5, 10]^T$ be a 4D column vector. What is magnitude of this vector when projected along the linear direction defined by the vector $\mathbf{v} = [0.6, 0, 0, 0.8]^T$?

Enter your answer here

Save Answer

Q3.4

1 Point

Let Q be a $d \times d$ orthogonal matrix and let \mathbf{x} be a d -dimensional column vector.

If we define the function g to be $g(\mathbf{x}) = Q\mathbf{x}$, what is the inverse of g (i.e., what is $g^{-1}(\mathbf{x})$)?

Enter your answer here

Save Answer

Q3.5

1 Point

What is the ℓ_2 norm of the vector $\mathbf{x} = [3, 4]$ (i.e., what is $\|\mathbf{x}\|_2$)?

Enter your answer here

Save Answer

Q3.6

1 Point

Suppose the singular value decomposition of a matrix A is USV^T where $S = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}$, what is the rank of A ?

Enter your answer here

Save Answer

Q3.7

1 Point

Suppose $A \in \mathbb{R}^{2 \times 3}$, $B \in \mathbb{R}^{3 \times 2}$ and $C \in \mathbb{R}^{3 \times 4}$, which of the following matrix multiplications operations are valid?

AB

BA

AC

CA

ABC

Save Answer

Q4 Programming
7 Points

Q4.1
1 Point

The following function is an attempt to compute the average value and averaged squared value of a list of numbers. However, there are two bugs, can you find the two bugs and correct the code?

```
def avg_and_squared_avg(x):  
    sum = 0  
    for i in range(len(x)-1):  
        sum = sum + x[i]  
    avg = sum/len(x)  
    for a in x:  
        sum += a*a  
    squared_avg = sum/len(x)  
    return avg, squared_avg
```

For example, if we call this function as follows:

`avg_and_squared_avg([1, 2, 3, 4])` the output is (1.5, 9.0) but the expected output is (2.5, 7.5)

Enter your answer here

Save Answer

Q4.2
1 Point

Write a function that takes a positive integer as input and returns a boolean indicating whether the number is prime or not. Do not worry much about efficiency. [Hint: Check if the number is divisible by any integer smaller than it but greater than 1.]

Enter your answer here

Save Answer

Q4.3
1 Point

Write a function that takes a list of integers as input and returns a list of the numbers in the list that are divisible by 2 and 3. For example, `get_divisible_by_2_and_3([2,6,12,5,60])` would return `[6,12,60]`.

Enter your answer here

Save Answer

Q4.4
1 Point

Using the documentation for numpy (<https://numpy.org/doc/stable/index.html>) and numpy broadcasting (<https://numpy.org/doc/stable/user/basics.broadcasting.html>), determine which of the following operations would be valid (i.e., not produce an error), where we assume that the shape of each numpy array is as follows:

- a has shape (2,)
- b has shape (3,2)
- c has shape (4,1,2)
- d has shape (1,5,2)

a * c

b * c

b * d

c * d

Save Answer

Q4.5

1 Point

Suppose you have Python lists of first names and last names called `first_names` and `last_names` respectively. Write a loop with a single statement that prints the first name, a space, and the corresponding last name without using indexes (i.e., only using a for loop with iterators and the `zip` function (<https://docs.python.org/3.3/library/functions.html#zip>)).

Enter your answer here

Save Answer

Q4.6

1 Point

What will the following code print:

```
x = 2
def foo(a):
    x = 5
    y = x*a
    return x
y = foo(2)
print(x,y)
```

Enter your answer here

Save Answer

Q4.7

1 Point

(Bonus, harder) Write a histogram function (see <https://en.wikipedia.org/wiki/Histogram>) in Python where the first argument is the data as a list of numbers and the second argument is the number of bins. The minimum and maximum values are assumed to be 0 and 1 (i.e., 0 and 1 should be the first and last bin edge). You can assume data only contains numbers between 0 and 1.

The output should be a list of counts for each bin (i.e., the number of data points that fall into each bin) and the bin edges. Do not worry about efficiency at this point.

For example, `histogram([0.1, 0.55, 0.85, 0.15], 5)`, the output should be `([2, 0, 1, 0, 1], [0, 0.2, 0.4, 0.6, 0.8, 1.0])`.

Enter your answer here

Save Answer