Why convolutions?

Neuroscientific inspiration

Computational reasons

1. Sparse computation (compared to full deep networks)
2. Shared parameters (only a small number of shared parameters)
3. Translation invariance

1D convolutions, similar but slightly different than signal processing / math convolutions ¶

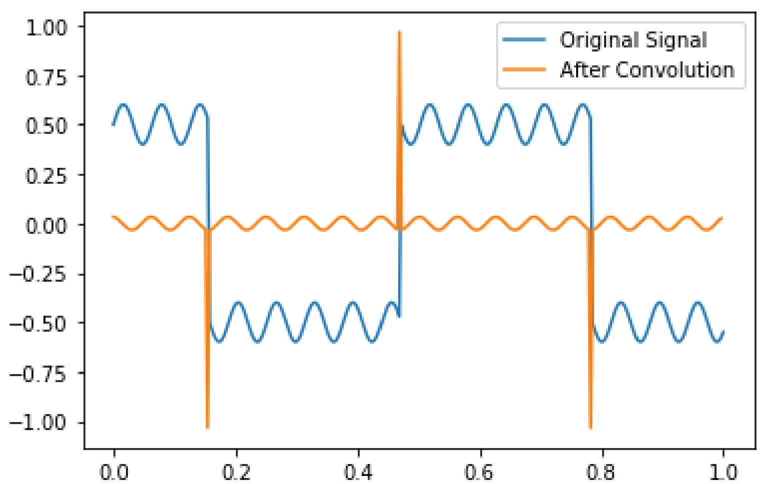
(Show on board, x signal, f is filter/kernel)

[-1, 1] filter/kernel highlights "sharp points" of signal

In [1]:
```
import torch
import matplotlib.pyplot as plt
%matplotlib inline

t = torch.linspace(0, 1.0, 300)
x = (torch.cos(10*t) > 0.0).float() + 0.1*torch.sin(100*t)-0.5
plt.plot(t.numpy(), x.numpy(), label='Original Signal')

from torch.nn import functional as F
filt = torch.tensor([-1, 1.0])
print('Filter')
print(filt)
# Should have shape $(m, c, w)$ where m is minibatch size, c is # channe
ls and w is width
y = F.conv1d(x.reshape(1, 1, len(x)), filt.reshape(1, 1, len(filt))).squ
eeze_()
plt.plot(t.numpy()[:len(y)], y.numpy(), label='After Convolution')
plt.legend()
```

```
Filter
tensor([-1.,  1.])
```

Out[1]: <matplotlib.legend.Legend at 0x120895470>



Convolutions are linear operators (i.e., matrix multiplication) with shared parameters

In [2]:
```
x = torch.randn(10).float().requires_grad_(True)
filt = torch.tensor([-1, 1]).float()
#filt = torch.tensor([1, 2, 3, 4]).float()
y = F.conv1d(x.reshape(1, 1, len(x)), filt.reshape(1, 1, len(filt))).squ
eeze_()

def extract_jacobian(x, y):
    J = torch.zeros((len(y), len(x))).float()
    for i in range(len(y)):
        v = torch.zeros(len(y)).float()
        v[i] = 1
        if x.grad is not None:
            x.grad.zero_()
        y.backward(v, retain_graph=True)
        J[i, :] = x.grad
    return J

A = extract_jacobian(x, y)
print(A)
y2 = torch.matmul(A, x)
print(y)
print(y2)
print(y-y2)
```

```
tensor([[-1.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
        [ 0., -1.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
        [ 0.,  0., -1.,  1.,  0.,  0.,  0.,  0.,  0.,  0.],
        [ 0.,  0.,  0., -1.,  1.,  0.,  0.,  0.,  0.,  0.],
        [ 0.,  0.,  0.,  0., -1.,  1.,  0.,  0.,  0.,  0.],
        [ 0.,  0.,  0.,  0.,  0., -1.,  1.,  0.,  0.,  0.],
        [ 0.,  0.,  0.,  0.,  0.,  0., -1.,  1.,  0.,  0.],
        [ 0.,  0.,  0.,  0.,  0.,  0.,  0., -1.,  1.,  0.],
        [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0., -1.,  1.]])
tensor([-1.0849,  2.0652, -0.1494,  2.2425, -1.5926, -1.0508, -0.1540,
-0.0375,
         0.9914], grad_fn=<AsStridedBackward>)
tensor([-1.0849,  2.0652, -0.1494,  2.2425, -1.5926, -1.0508, -0.1540,
-0.0375,
         0.9914], grad_fn=<MvBackward>)
tensor([0., 0., 0., 0., 0., 0., 0., 0., 0.], grad_fn=<SubBackward0>)
```

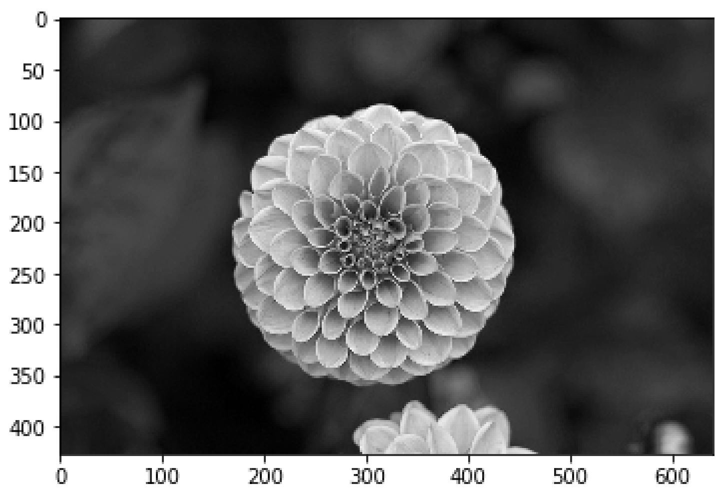
Side note: Matrices that have constants along diagonals are called "Toeplitz Matrices"

$$\begin{bmatrix} a & b & c & d & e \\ f & a & b & c & d \\ g & f & a & b & c \\ h & g & f & a & b \\ i & h & g & f & a \end{bmatrix}$$

2D convolutions are similar and can be applied to images

In [3]:
```
import sklearn.datasets
A = torch.tensor(sklearn.datasets.load_sample_image('china.jpg')).float
()
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float
()
A = torch.sum(A, dim=2)
plt.imshow(A, cmap='gray')
```

Out[3]:
```
<matplotlib.image.AxesImage at 0x10a13a240>
```

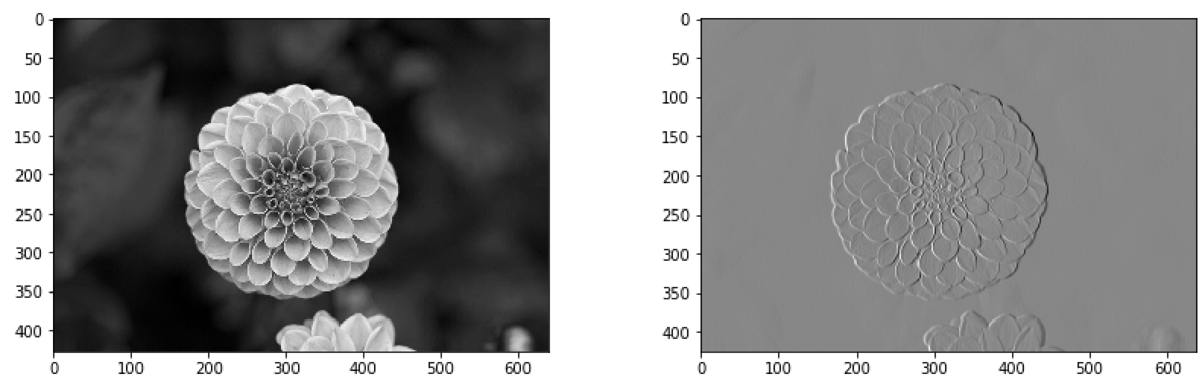


Different filters extract different features from the image

In [4]:
```
filt = torch.tensor([[-1, 0, 1], [-1, 0, 1], [-1, 0, 1]]).float() # Hori
zontal
#filt = torch.tensor([[-1, 0, 1], [-1, 0, 1], [-1, 0, 1]]).float().t() #
Vertical
#filt = torch.tensor([[1, -1], [-1, 1]]).float() # Checker board pattern
#filt = torch.ones((10, 10)).float() # Blur
print('Filter')
print(filt)
B = F.conv2d(A.reshape(1, 1, *A.size()), filt.reshape(1, 1, *filt.size
()), padding=1).squeeze()
B = F.conv2d(A.reshape(1, 1, *A.size()), filt.reshape(1, 1, *filt.size
())).squeeze()
print('A size', A.size(), 'B size', B.size())

fig, axes = plt.subplots(1, 2, figsize=(14,4))
axes[0].imshow(A.numpy(), cmap='gray')
axes[1].imshow(B.numpy(), cmap='gray')
```

```
Filter
tensor([[-1.,  0.,  1.],
        [-1.,  0.,  1.],
        [-1.,  0.,  1.]])
A size torch.Size([427, 640]) B size torch.Size([425, 638])
```

Out[4]: <matplotlib.image.AxesImage at 0x1a239b26d8>



Higher dimensional convolutions are similar (i.e., if there is more than 1 channel)

(Show on board)

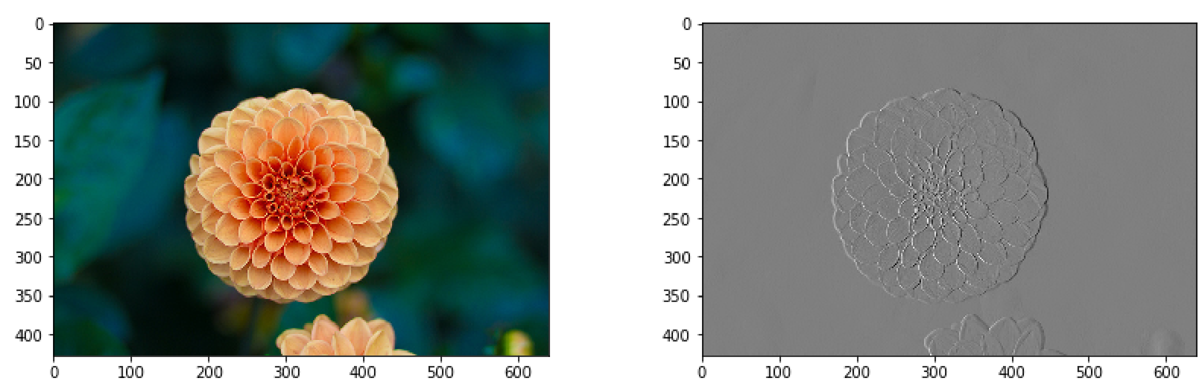
In [5]:

```
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float
()
A = A/255
A = A.permute(2,0,1)
print(A.size())

filt = torch.tensor([1, 0, 0]).reshape(3, 1, 1).float() # Extract red
#filt = torch.tensor([0, 1, 0]).reshape(3, 1, 1).float() # Extract green
#filt = torch.tensor([0, 0, 1]).reshape(3, 1, 1).float() # Extract blue
filt = torch.ones(3, 5, 5).float() # Blur
filt = torch.tensor([
    [[-1, 1]],
    [[-1, 1]],
    [[-1, 1]],
]).float()

print('Filter')
print(filt)
print(filt.size())
B = F.conv2d(A.reshape(1, *A.size()), filt.reshape(1, *filt.size()), pad
ding=1).squeeze()
B = F.conv2d(A.reshape(1, *A.size()), filt.reshape(1, *filt.size())).squ
eeze()
print('A size', A.size(), 'B size', B.size())

fig, axes = plt.subplots(1, 2, figsize=(14,4))
axes[0].imshow(A.permute(1,2,0), cmap='gray')
axes[1].imshow(B, cmap='gray')
```

```
torch.Size([3, 427, 640])
Filter
tensor([[[[-1.,  1.]],

         [[-1.,  1.]],

         [[-1.,  1.]]]])
torch.Size([3, 1, 2])
A size torch.Size([3, 427, 640]) B size torch.Size([427, 639])
```

Out[5]:

```
<matplotlib.image.AxesImage at 0x10a7d6da0>
```



How to interpret convolution descriptions (usually)

Kernel sizes assume all channels (e.g., "1x1 convolution" corresponds to a kernel size of 1x1xC where C is the number of channles)

The number of filters in the previous layer corresponds to the number of channels in the curent layer

(Show on board)

Why convolutions again?

Computational reasons

1. Sparse computation (compared to full deep networks)
2. Shared parameters (only a small number of shared parameters)
3. Translation invariance

Extract image features (edges, etc.)

Automatically learn image features

Why activation functions? Activation functions enable non-linear models

Consider a deep linear network

In [6]:
```python
torch.manual_seed(0)
A1 = torch.randn((10, 5))
A2 = torch.randn((10, 10))
A3 = torch.randn((1, 10))

x = torch.randn(5)
print('x', x)
y = torch.matmul(A1, x)
y = torch.matmul(A2, y)
y = torch.matmul(A3, y)
print('y', y)

b = torch.matmul(A3, torch.matmul(A2, A1))
y2 = torch.matmul(b, x)
print('y2', y2)
```

```
x tensor([ 1.4875, -0.2230, -1.0057, -0.4139,  1.1600])
y tensor([4.1752])
y2 tensor([4.1752])
```

If you add activation functions, the deep function cannot be simplified

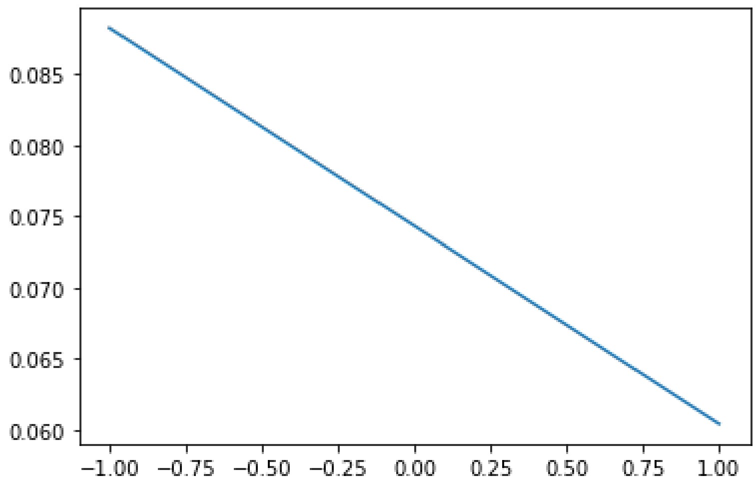
In [7]:
```python
torch.manual_seed(0)
A1 = torch.randn((10, 5))
A2 = torch.randn((10, 10))
A3 = torch.randn((1, 10))

x = torch.randn(5)
print('x', x)
y = torch.matmul(A1, x)
y = torch.relu(y)
y = torch.matmul(A2, y)
y = torch.relu(y)
y = torch.matmul(A3, y)
print('y', y)

b = torch.matmul(A3, torch.matmul(A2, A1))
y2 = torch.matmul(b, x)
print('y2', y2)
```

```
x tensor([ 1.4875, -0.2230, -1.0057, -0.4139,  1.1600])
y tensor([18.9449])
y2 tensor([4.1752])
```

Without ReLU or activation function, the function can only be linear

In [8]:
```
N, D_in, H, D_out = 64, 1, 10, 1
model = torch.nn.Sequential(
    torch.nn.Linear(D_in, H),
    torch.nn.Linear(H, D_out),
)
x = torch.linspace(-1, 1).reshape(-1, 1)
y = model(x)
plt.plot(x.detach().numpy(), y.detach().numpy())
```

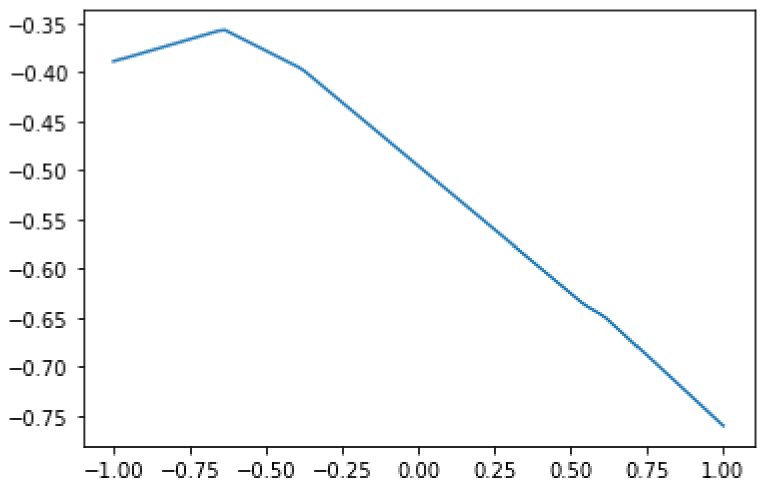
Out[8]:
```
[<matplotlib.lines.Line2D at 0x1a236c1208>]
```



With ReLU activation function, the function is *piecewise* linear

In [9]:
```
N, D_in, H, D_out = 64, 1, 10, 1
model = torch.nn.Sequential(
    torch.nn.Linear(D_in, H),
    torch.nn.ReLU(),
    torch.nn.Linear(H, D_out),
)
x = torch.linspace(-1, 1).reshape(-1, 1)
y = model(x)
plt.plot(x.detach().numpy(), y.detach().numpy())
```

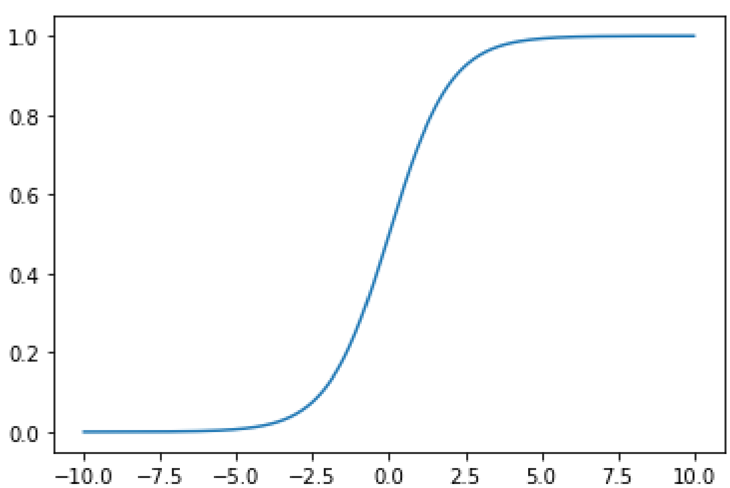
Out[9]:
```
[<matplotlib.lines.Line2D at 0x10a873c50>]
```



Common activation functions include sigmoid, ReLU, Leaky ReLU, tanh

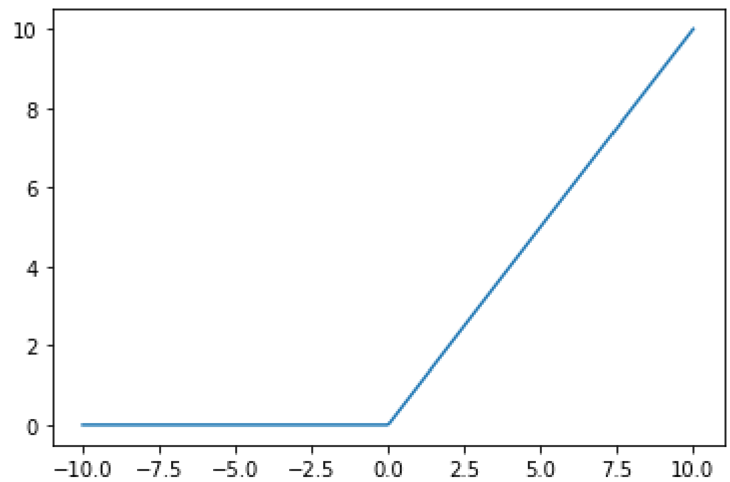
In [10]:
```
t = torch.linspace(-10, 10, 300)
plt.plot(t.numpy(), torch.sigmoid(t).numpy())
```

Out[10]:
```
[<matplotlib.lines.Line2D at 0x1a2306f470>]
```



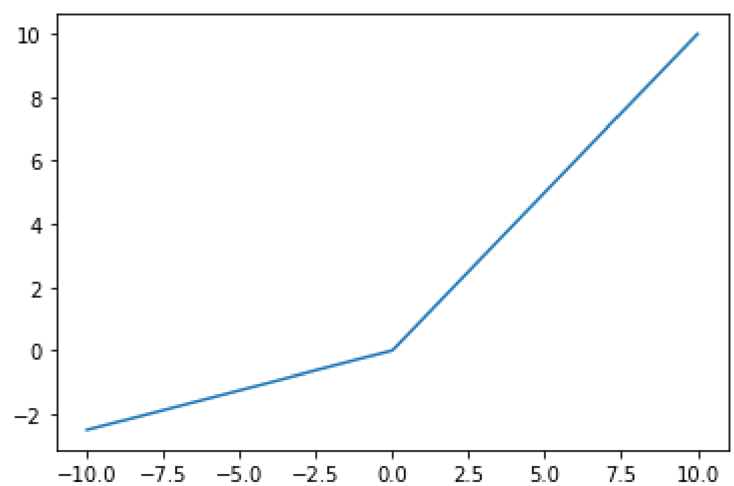
In [11]:
```
plt.plot(t.numpy(), F.relu(t).numpy())
```

Out[11]:
```
[<matplotlib.lines.Line2D at 0x1a2359d6a0>]
```



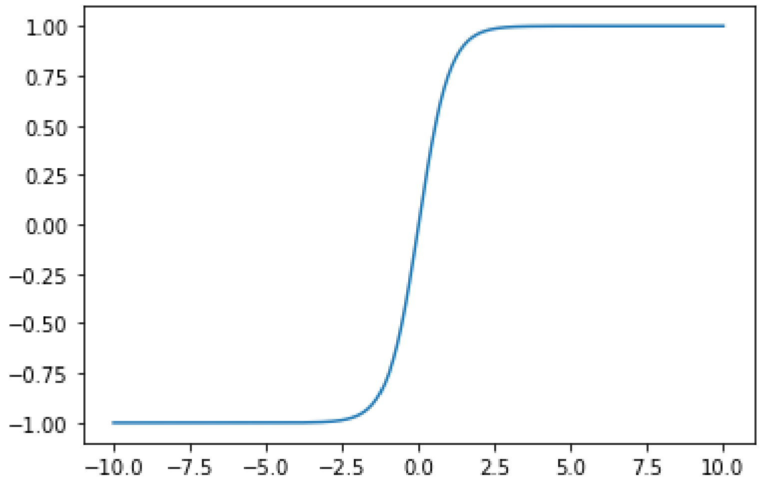
In [12]:
```
plt.plot(t.numpy(), F.leaky_relu(t, negative_slope=0.25).numpy())
```

Out[12]:
```
[<matplotlib.lines.Line2D at 0x1a23a0ba20>]
```



In [13]:
```
plt.plot(t.numpy(), torch.tanh(t).numpy())
```

Out[13]: [<matplotlib.lines.Line2D at 0x1a24274208>]



Pooling layers are used to reduce dimensionality and introduce some location invariance

Pooling layers include max pooling and average pooling

In [14]:
```
torch.manual_seed(0)
x = torch.randint(10, (10,)).float()
y = F.max_pool1d(x.reshape(1,1,-1), kernel_size=3)
y2 = F.max_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1)
y3 = F.max_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1, padding=1)
#y = F.avg_pool1d(x.reshape(1,1,-1), kernel_size=3)
#y2 = F.avg_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1)
#y3 = F.avg_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1, padding=
1)
print(x)
print(y)
print(y2)
print(y3)
```

```
tensor([4., 9., 3., 0., 3., 9., 7., 3., 7., 3.])
tensor([[[9., 9., 7.]]])
tensor([[[9., 9., 3., 9., 9., 9., 7., 7.]]])
tensor([[[9., 9., 9., 3., 9., 9., 9., 7., 7., 7.]]])
```

Is average pooling a linear or non-linear operation?

Is max pooling a linear or non-linear operation?

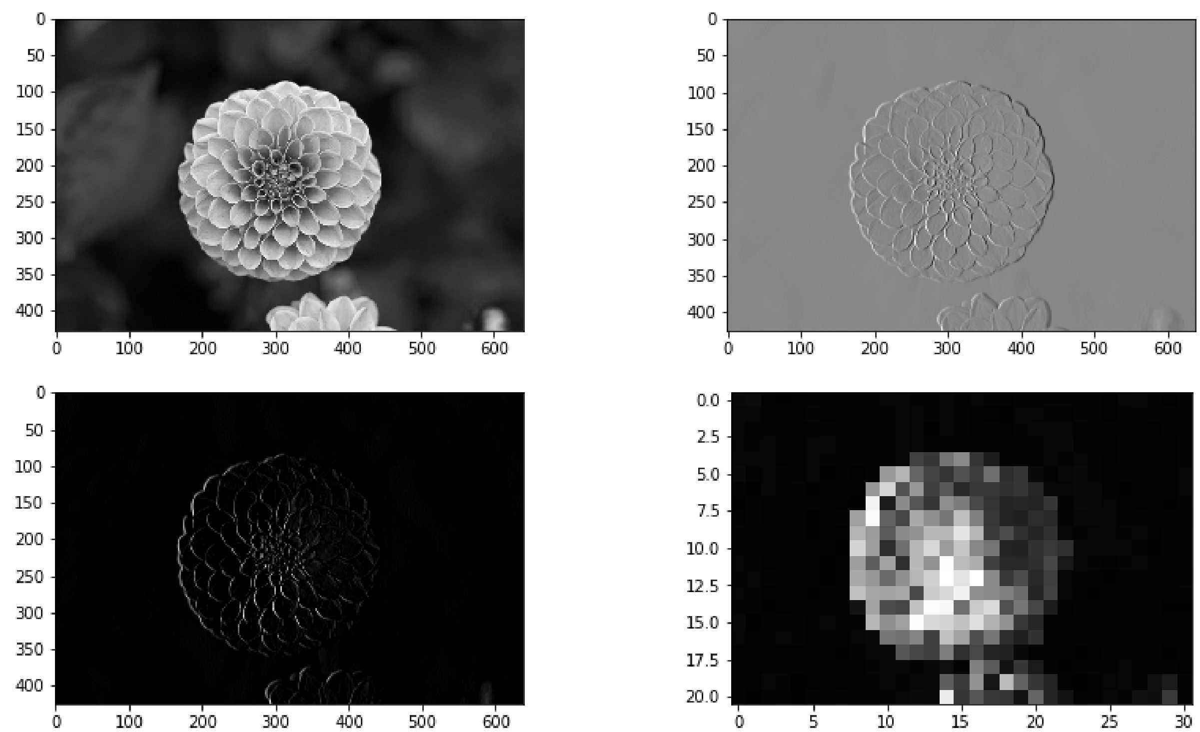
Convolution Neural Network (CNN) layers are compositions of convolution, activation and pooling

(See illustration on slide)

In [16]:

```
import sklearn.datasets
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float
()
A = torch.sum(A, dim=2)
filt = torch.tensor([[-1, 0, 1], [-1, 0, 1], [-1, 0, 1]]).float() # Hori
zontal
#filt = torch.tensor([[-1, 0, 1], [-1, 0, 1], [-1, 0, 1]]).float().t() #
Vertical
#filt = torch.tensor([[1, -1], [-1, 1]]).float() # Checker board pattern
#filt = torch.ones((10, 10)).float() # Blur
print('Filter')
print(filt)
B = F.conv2d(A.reshape(1, 1, *A.size()), filt.reshape(1, 1, *filt.size
()))
print('A size', A.size(), 'B size', B.size())
C = torch.relu(B)
D = torch.max_pool2d(C, kernel_size=20)
#D = torch.max_pool2d(C, kernel_size=20, stride=1)

fig, axes = plt.subplots(2, 2, figsize=(14,8))
axes = axes.ravel()
for im, ax in zip([A, B, C, D], axes):
    ax.imshow(im.squeeze(), cmap='gray')
```

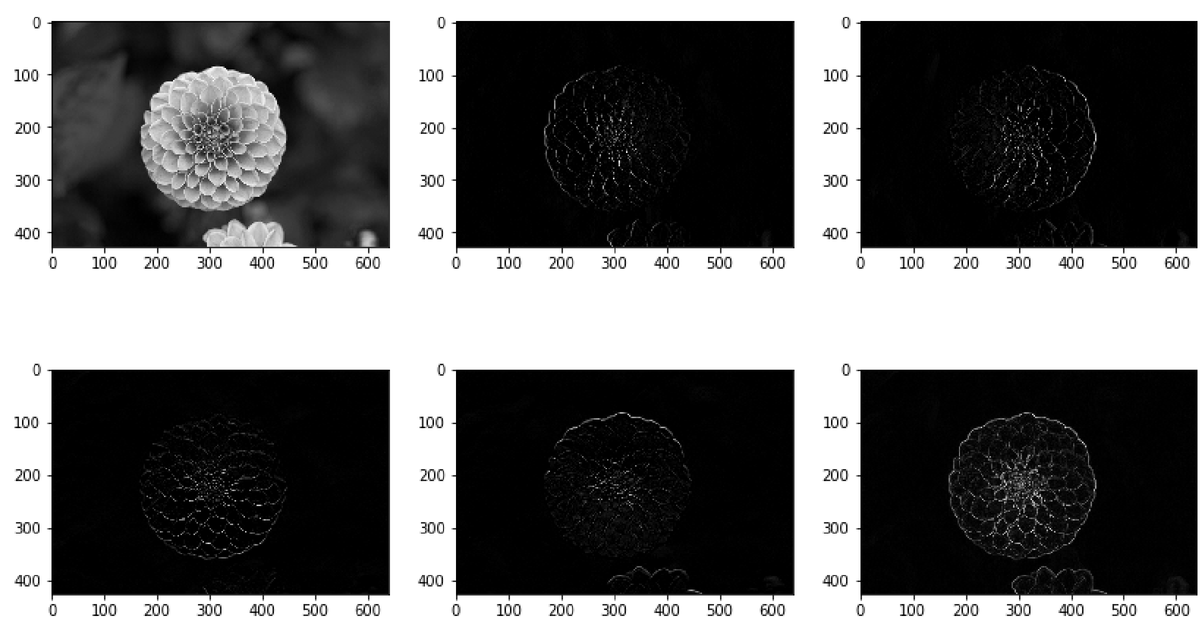
```
Filter
tensor([[-1.,  0.,  1.],
        [-1.,  0.,  1.],
        [-1.,  0.,  1.]])
A size torch.Size([427, 640]) B size torch.Size([1, 1, 425, 638])
```



How could you detect an edge from multiple angles by combining convolutions and ReLUs?

Hint: First detect edges from all directions, then combine.

In [25]:
```
import sklearn.datasets
import torch
import numpy as np
A = torch.tensor(sklearn.datasets.load_sample_image('china.jpg')).float
()
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float
()
A = torch.sum(A, dim=2)

filters = torch.tensor([
    [[[-1, 1], [-1, 1]]],
    [[[1, -1], [1, -1]]],
    [[[1, 1], [-1, -1]]],
    [[[-1, -1], [1, 1]]],
]).float()
B = F.conv2d(A.reshape(1, 1, *A.size()), filters)
C = torch.relu(B)

# Combine
filt = torch.ones(4).float()
D = F.conv2d(C, filt.reshape(1, 4, 1, 1))

fig, axes = plt.subplots(2, 3, figsize=(14,8))
for im, ax in zip([A, *C[0,:,:,:], D], axes.ravel()):
    ax.imshow(im.squeeze(), cmap='gray')
```



Check out PyTorch tutorial on simple classifier on CIFAR10 dataset:

https://pytorch.org/tutorials/beginner/blitz/cifar10_tutorial.html
(https://pytorch.org/tutorials/beginner/blitz/cifar10_tutorial.html)