Why convolutions?

Neuroscientific inspiration

Computational reasons

1. Sparse computation (compared to full deep networks)
2. Shared parameters (only a small number of shared parameters)
3. Translation invariance

1D convolutions, similar but slightly different than signal processing / math convolutions

(Show on board, x signal, f is filter/kernel)

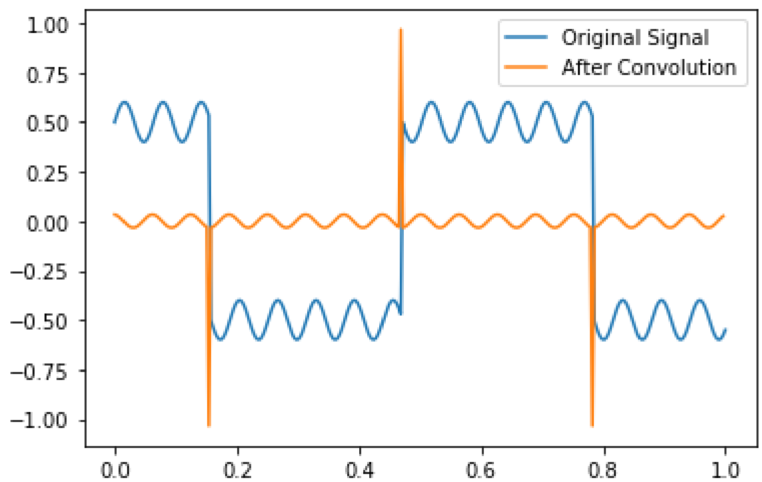
[-1, 1] filter/kernel highlights "sharp points" of signal

In [1]:

```
import torch
import matplotlib.pyplot as plt
%matplotlib inline

t = torch.linspace(0, 1.0, 300)
x = (torch.cos(10*t) > 0.0).float() + 0.1*torch.sin(100*t)-0.5
plt.plot(t.numpy(), x.numpy(), label='Original Signal')

from torch.nn import functional as F
filt = torch.tensor([-1, 1.0])
print('Filter')
print(filt)
# Should have shape $(m, c, w)$ where m is minibatch size, c is # channels
y = F.conv1d(x.reshape(1, 1, len(x)), filt.reshape(1, 1, len(filt))).squeez
plt.plot(t.numpy()[:len(y)], y.numpy(), label='After Convolution')
plt.legend()
```

```
Filter
tensor([-1.,  1.])
```

Out[1]: <matplotlib.legend.Legend at 0x12233f860>



Convolutions are linear operators (i.e., matrix multiplication) with shared parameters

In [2]:
```python
x = torch.randn(10).float().requires_grad_(True)
filt = torch.tensor([-1, 1]).float()
#filt = torch.tensor([1, 2, 3, 4]).float()
y = F.conv1d(x.reshape(1, 1, len(x)), filt.reshape(1, 1, len(filt))).squeez

def extract_jacobian(x, y):
    J = torch.zeros((len(y), len(x))).float()
    for i in range(len(y)):
        v = torch.zeros(len(y)).float()
        v[i] = 1
        if x.grad is not None:
            x.grad.zero_()
        y.backward(v, retain_graph=True)
        J[i, :] = x.grad
    return J

A = extract_jacobian(x, y)
print(A)
y2 = torch.matmul(A, x)
print(y)
print(y2)
print(y-y2)
```

```
tensor([[-1.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
        [ 0., -1.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
        [ 0.,  0., -1.,  1.,  0.,  0.,  0.,  0.,  0.,  0.],
        [ 0.,  0.,  0., -1.,  1.,  0.,  0.,  0.,  0.,  0.],
        [ 0.,  0.,  0.,  0., -1.,  1.,  0.,  0.,  0.,  0.],
        [ 0.,  0.,  0.,  0.,  0., -1.,  1.,  0.,  0.,  0.],
        [ 0.,  0.,  0.,  0.,  0.,  0., -1.,  1.,  0.,  0.],
        [ 0.,  0.,  0.,  0.,  0.,  0.,  0., -1.,  1.,  0.],
        [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0., -1.,  1.]])
tensor([ 0.2579,  0.1628,  0.4810, -2.4788,  2.1865, -0.0637, -2.1996,
0.5330,
         1.9802], grad_fn=<AsStridedBackward>)
tensor([ 0.2579,  0.1628,  0.4810, -2.4788,  2.1865, -0.0637, -2.1996,
0.5330,
         1.9802], grad_fn=<MvBackward>)
tensor([0., 0., 0., 0., 0., 0., 0., 0., 0.], grad_fn=<SubBackward0>)
```

2D convolutions are similar and can be applied to images

(Show on board, x is 2D image, f is 2D kernel)

Different filters extract different features from the image

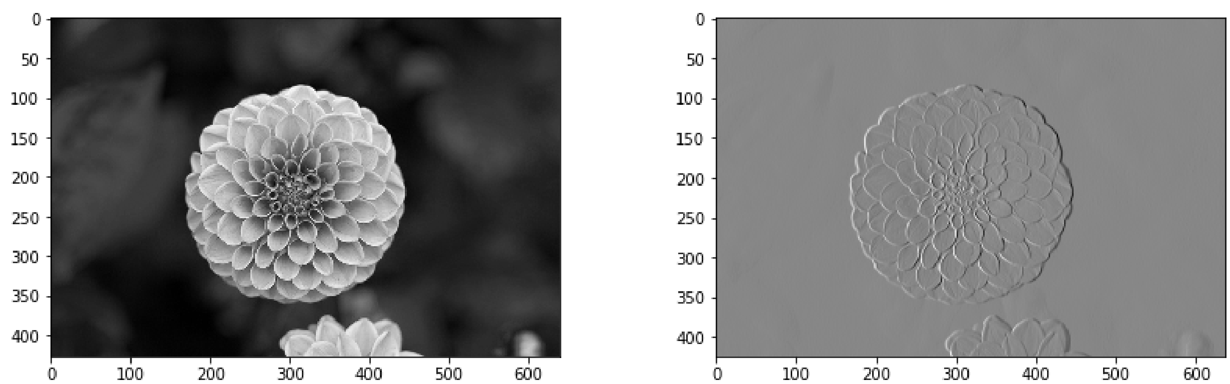
In [16]:

```python
import sklearn.datasets
A = torch.tensor(sklearn.datasets.load_sample_image('china.jpg')).float()
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float()
A = torch.sum(A, dim=2) # Sum channels

filt = torch.tensor([[-1, 0, 1], [-1, 0, 1], [-1, 0, 1]]).float() # Horizon
#filt = torch.tensor([[-1, 0, 1], [-1, 0, 1], [-1, 0, 1]]).float().t() # Ve
#filt = torch.tensor([[1, -1], [-1, 1]]).float() # Checker board pattern
#filt = torch.ones((10, 10)).float() # Blur
print('Filter')
print(filt)
B = F.conv2d(A.reshape(1, 1, *A.size()), filt.reshape(1, 1, *filt.size()),
B = F.conv2d(A.reshape(1, 1, *A.size()), filt.reshape(1, 1, *filt.size())).
print('A size', A.size(), 'B size', B.size())

fig, axes = plt.subplots(1, 2, figsize=(14,4))
axes[0].imshow(A.numpy(), cmap='gray')
axes[1].imshow(B.numpy(), cmap='gray')
```

```
Filter
tensor([[-1.,  0.,  1.],
        [-1.,  0.,  1.],
        [-1.,  0.,  1.]])
A size torch.Size([427, 640]) B size torch.Size([425, 638])
```

Out[16]:

```
<matplotlib.image.AxesImage at 0x1a2596fe48>
```



Higher dimensional convolutions are similar (i.e., if there is more than 1 channel)

(Show higher dimensional convolution on board)

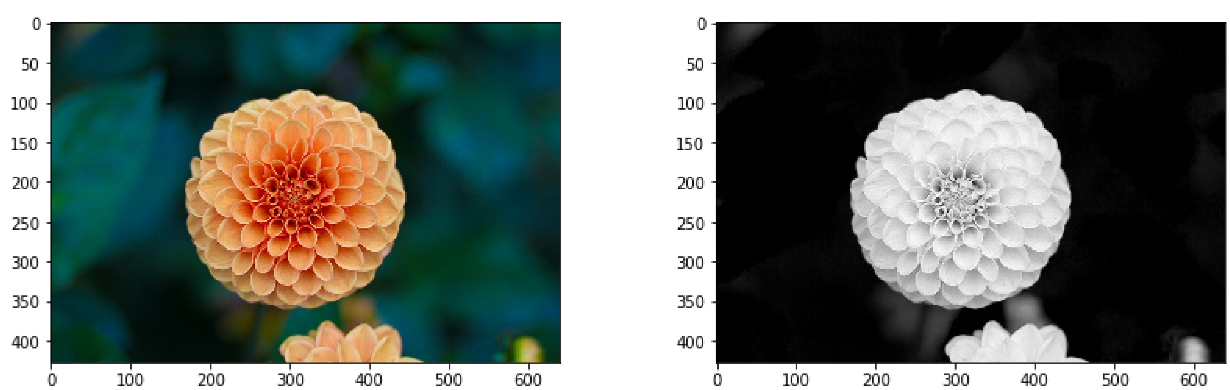
In [18]:

```
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float()
A = A/255
A = A.permute(2,0,1)
print(A.size())

filt = torch.tensor([1, 0, 0]).reshape(3, 1, 1).float() # Extract red
#filt = torch.tensor([0, 1, 0]).reshape(3, 1, 1).float() # Extract green
#filt = torch.tensor([0, 0, 1]).reshape(3, 1, 1).float() # Extract blue
#filt = torch.ones(3, 5, 5).float() # Blur
#filt = torch.tensor([
#    [[-1, 1]],
#    [[-1, 1]],
#    [[-1, 1]],
#]).float()

print('Filter')
print(filt)
print(filt.size())
B = F.conv2d(A.reshape(1, *A.size()), filt.reshape(1, *filt.size())).squeez
print('A size', A.size(), 'B size', B.size())

fig, axes = plt.subplots(1, 2, figsize=(14,4))
axes[0].imshow(A.permute(1,2,0), cmap='gray')
axes[1].imshow(B, cmap='gray')
```

```
torch.Size([3, 427, 640])
Filter
tensor([[[1.]],

        [[0.]],

        [[0.]]])
torch.Size([3, 1, 1])
A size torch.Size([3, 427, 640]) B size torch.Size([427, 640])
```

Out[18]:

```
<matplotlib.image.AxesImage at 0x1a25ecae48>
```



How to interpret convolution descriptions (usually)

Kernel sizes assume all channels (e.g., "1x1 convolution" corresponds to a kernel size of 1x1xC where C is the number of channles)

The number of filters in the previous layer corresponds to the number of channels in the curent layer

Why convolutions again?

Computational reasons

1. Sparse computation (compared to full deep networks)
2. Shared parameters (only a small number of shared parameters)
3. Translation invariance

Extract image features (edges, etc.)

Automatically learn image features

Need several other components for extracting features: Activation functions and pooling layers

Why activation functions? Activation functions enable non-linear models

Consider a deep linear network

In [5]:
```
torch.manual_seed(0)
A1 = torch.randn((10, 5))
A2 = torch.randn((10, 10))
A3 = torch.randn((1, 10))

x = torch.randn(5)
print('x', x)
y = torch.matmul(A1, x)
y = torch.matmul(A2, y)
y = torch.matmul(A3, y)
print('y', y)

b = torch.matmul(A3, torch.matmul(A2, A1))
y2 = torch.matmul(b, x)
print('y2', y2)
```

```
x tensor([ 1.4875, -0.2230, -1.0057, -0.4139,  1.1600])
y tensor([4.1752])
y2 tensor([4.1752])
```

If you add activation functions, the deep function cannot be simplified

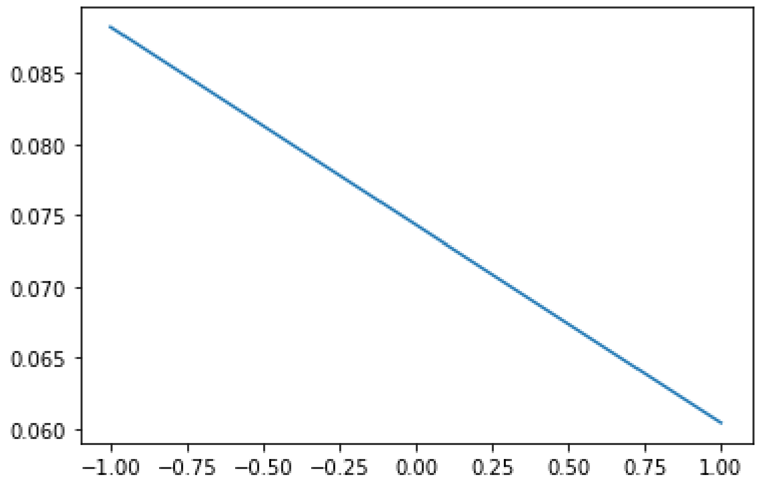
In [6]:
```
torch.manual_seed(0)
A1 = torch.randn((10, 5))
A2 = torch.randn((10, 10))
A3 = torch.randn((1, 10))

x = torch.randn(5)
print('x', x)
y = torch.matmul(A1, x)
y = torch.relu(y)
y = torch.matmul(A2, y)
y = torch.relu(y)
y = torch.matmul(A3, y)
print('y', y)

b = torch.matmul(A3, torch.matmul(A2, A1))
y2 = torch.matmul(b, x)
print('y2', y2)
```

```
x tensor([ 1.4875, -0.2230, -1.0057, -0.4139,  1.1600])
y tensor([18.9449])
y2 tensor([4.1752])
```

Without ReLU or activation function, the function can only be linear

In [7]:
```
N, D_in, H, D_out = 64, 1, 10, 1
model = torch.nn.Sequential(
    torch.nn.Linear(D_in, H),
    torch.nn.Linear(H, D_out),
)
x = torch.linspace(-1, 1).reshape(-1, 1)
y = model(x)
plt.plot(x.detach().numpy(), y.detach().numpy())
```

Out[7]:
```
[<matplotlib.lines.Line2D at 0x1a24ea1828>]
```



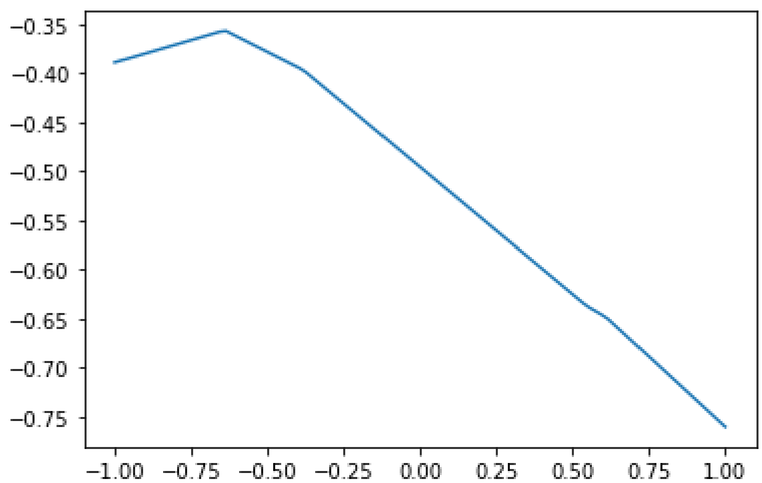
With ReLU activation function, the function is *piecewise* linear

In [8]:

```
N, D_in, H, D_out = 64, 1, 10, 1
model = torch.nn.Sequential(
    torch.nn.Linear(D_in, H),
    torch.nn.ReLU(),
    torch.nn.Linear(H, D_out),
)
x = torch.linspace(-1, 1).reshape(-1, 1)
y = model(x)
plt.plot(x.detach().numpy(), y.detach().numpy())
```

Out[8]:

```
[<matplotlib.lines.Line2D at 0x1223e9828>]
```



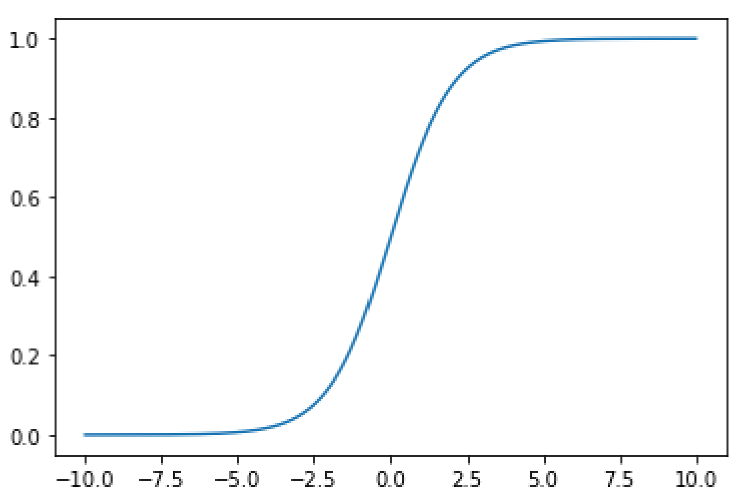
Common activation functions include sigmoid, ReLU, Leaky ReLU, tanh

In [9]:

```
t = torch.linspace(-10, 10, 300)
plt.plot(t.numpy(), torch.sigmoid(t).numpy())
```

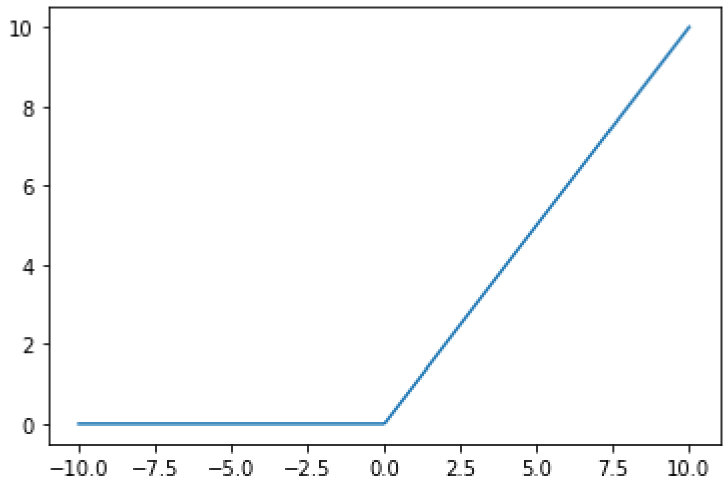
Out[9]:

```
[<matplotlib.lines.Line2D at 0x1a25ccb160>]
```



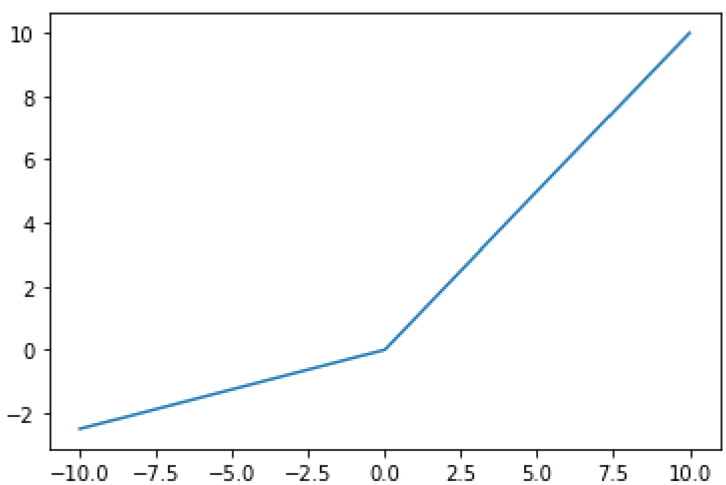
In [10]:
```
plt.plot(t.numpy(), F.relu(t).numpy())
```

Out[10]: [<matplotlib.lines.Line2D at 0x1a254bd208>]



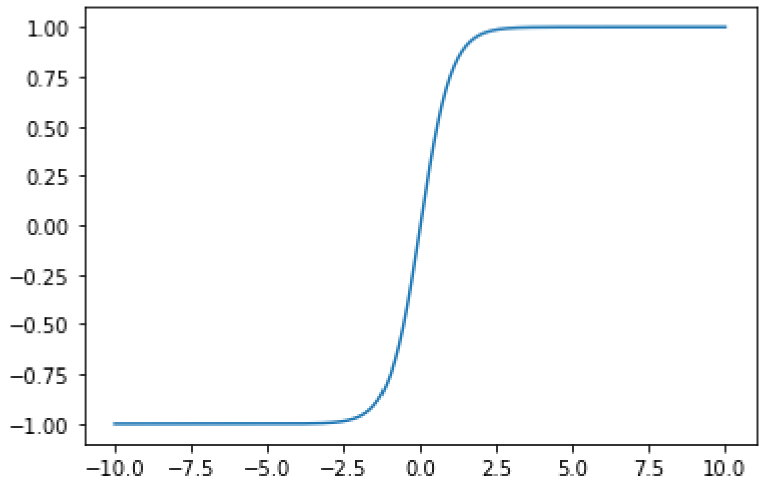
In [11]:
```
plt.plot(t.numpy(), F.leaky_relu(t, negative_slope=0.25).numpy())
```

Out[11]: [<matplotlib.lines.Line2D at 0x1a25445588>]



In [12]:
```
plt.plot(t.numpy(), torch.tanh(t).numpy())
```

Out[12]:
```
[<matplotlib.lines.Line2D at 0x1a253b3d30>]
```



Pooling layers are used to reduce dimensionality and introduce some location invariance

Pooling layers include max pooling and average pooling

In [13]:
```python
torch.manual_seed(0)
x = torch.randint(10, (10,)).float()
y = F.max_pool1d(x.reshape(1,1,-1), kernel_size=3)
y2 = F.max_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1)
y3 = F.max_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1, padding=1)
#y = F.avg_pool1d(x.reshape(1,1,-1), kernel_size=3)
#y2 = F.avg_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1)
#y3 = F.avg_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1, padding=1)
print(x)
print(y)
print(y2)
print(y3)
```

```
tensor([4., 9., 3., 0., 3., 9., 7., 3., 7., 3.])
tensor([[[9., 9., 7.]]])
tensor([[[9., 9., 3., 9., 9., 9., 7., 7.]]])
tensor([[[9., 9., 9., 3., 9., 9., 9., 7., 7., 7.]]])
```

Is average pooling a linear or non-linear operation?

Is max pooling a linear or non-linear operation?

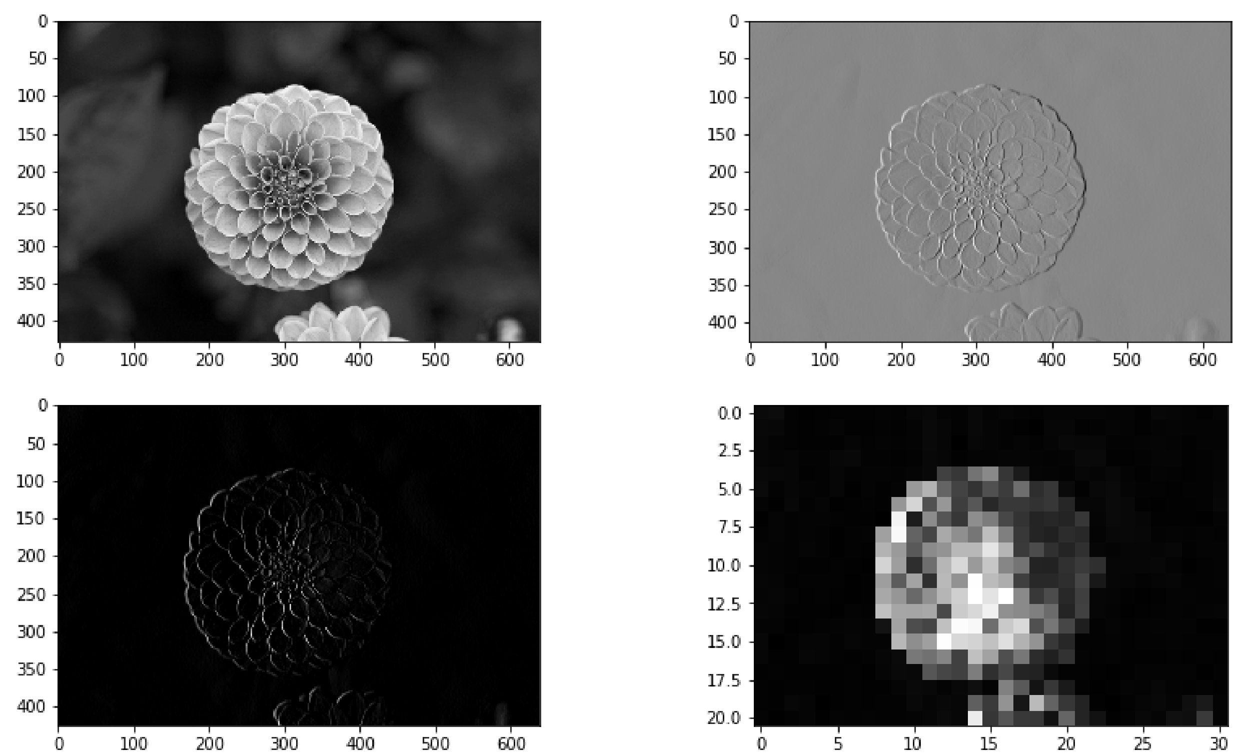
Convolution Neural Network (CNN) layers are compositions of convolution, activation and pooling

(See illustration on slide)

In [14]:

```
import sklearn.datasets
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float()
A = torch.sum(A, dim=2)
filt = torch.tensor([[-1, 0, 1], [-1, 0, 1], [-1, 0, 1]]).float() # Horizon
#filt = torch.tensor([[-1, 0, 1], [-1, 0, 1], [-1, 0, 1]]).float().t() # Ve
#filt = torch.tensor([[1, -1], [-1, 1]]).float() # Checker board pattern
#filt = torch.ones((10, 10)).float() # Blur
print('Filter')
print(filt)
B = F.conv2d(A.reshape(1, 1, *A.size()), filt.reshape(1, 1, *filt.size()))
print('A size', A.size(), 'B size', B.size())
C = torch.relu(B)
D = torch.max_pool2d(C, kernel_size=20)
#D = torch.max_pool2d(C, kernel_size=20, stride=1)

fig, axes = plt.subplots(2, 2, figsize=(14,8))
axes = axes.ravel()
for im, ax in zip([A, B, C, D], axes):
    ax.imshow(im.squeeze(), cmap='gray')
```

```
Filter
tensor([[-1.,  0.,  1.],
        [-1.,  0.,  1.],
        [-1.,  0.,  1.]])
A size torch.Size([427, 640]) B size torch.Size([1, 1, 425, 638])
```

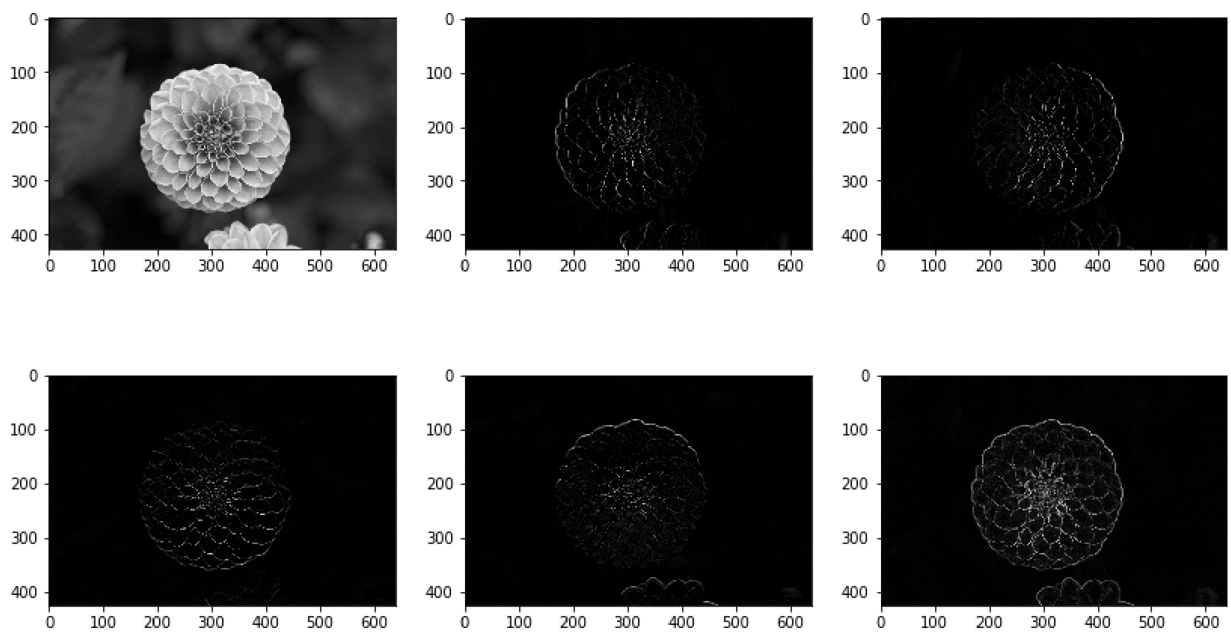


How could you detect an edge from multiple angles by combining convolutions and ReLUs?

Hint: First detect edges from all directions, then combine.

In [15]:

```python
import sklearn.datasets
import torch
import numpy as np
A = torch.tensor(sklearn.datasets.load_sample_image('china.jpg')).float()
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float()
A = torch.sum(A, dim=2)

filters = torch.tensor([
    [[[-1, 1], [-1, 1]]],
    [[[1, -1], [1, -1]]],
    [[[1, 1], [-1, -1]]],
    [[[-1, -1], [1, 1]]],
]).float()
B = F.conv2d(A.reshape(1, 1, *A.size()), filters)
C = torch.relu(B)

# Combine
filt = torch.ones(4).float()
D = F.conv2d(C, filt.reshape(1, 4, 1, 1))

fig, axes = plt.subplots(2, 3, figsize=(14,8))
for im, ax in zip([A, *C[0,:,:,:], D], axes.ravel()):
    ax.imshow(im.squeeze(), cmap='gray')
```



Check out PyTorch tutorial on simple classifier on CIFAR10 dataset:

[https://pytorch.org/tutorials/beginner/blitz/cifar10_tutorial.html (https://pytorch.org/tutorials/beginner/blitz/cifar10_tutorial.html)](https://pytorch.org/tutorials/beginner/blitz/cifar10_tutorial.html)