Convolutional Neural Networks (CNN)

ECE57000: Artificial Intelligence

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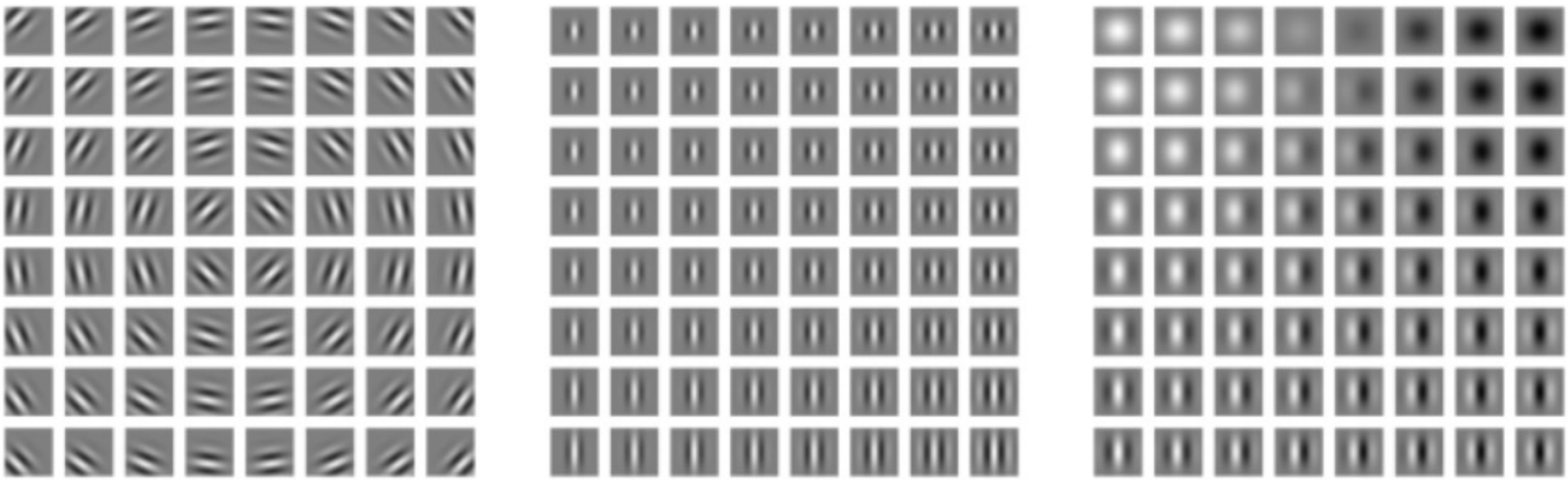
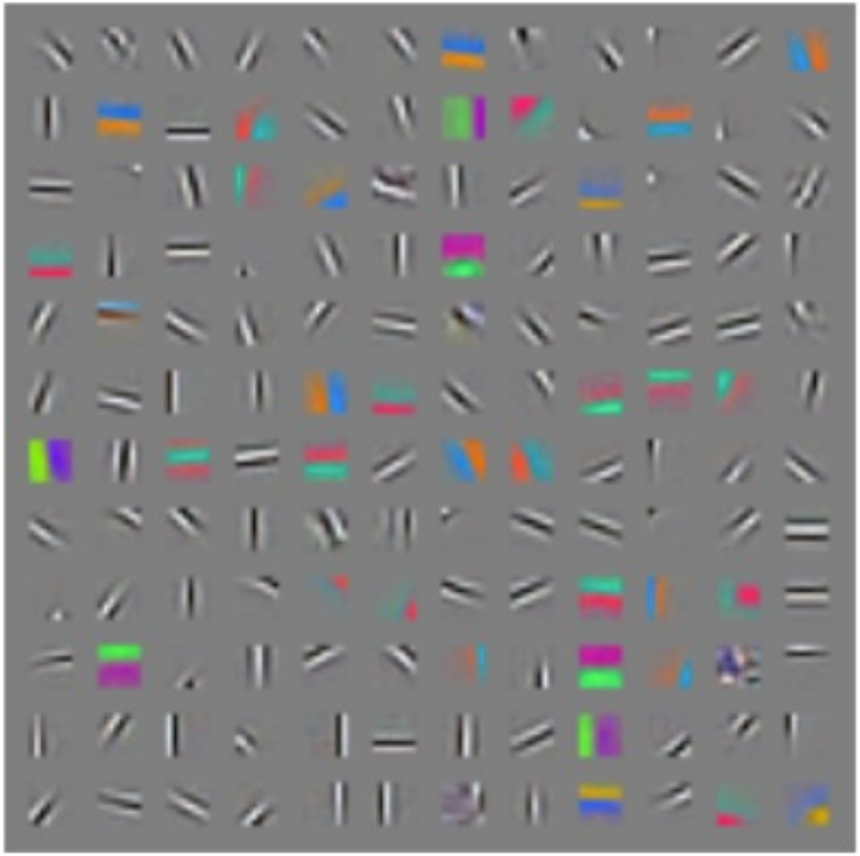
Why convolutional networks?

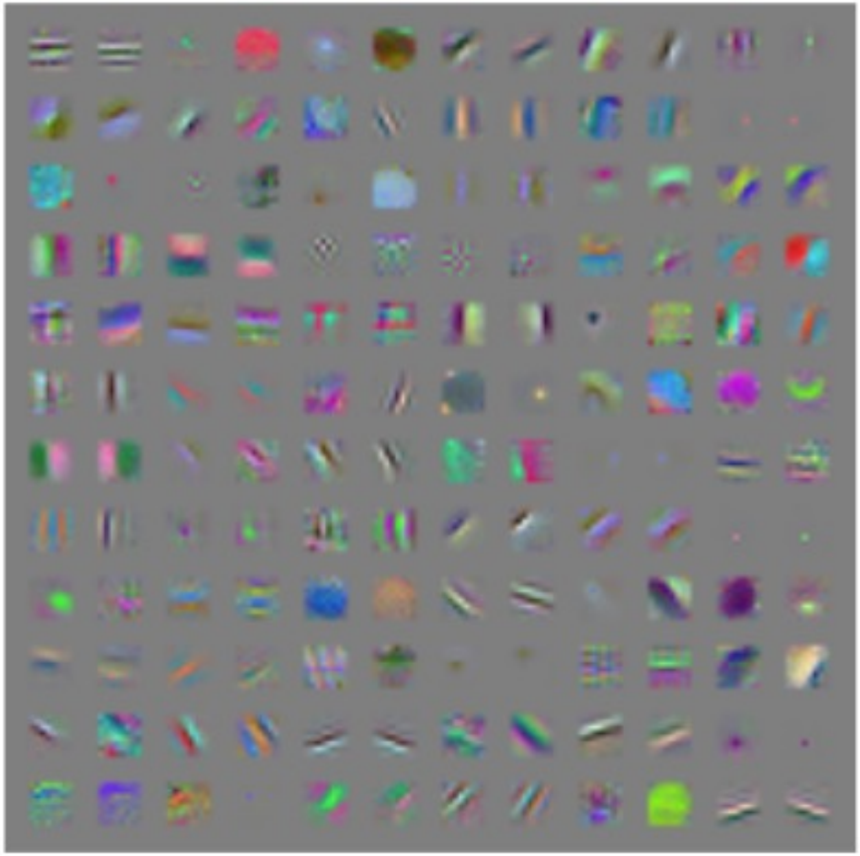
- Neuroscientific inspiration
- Computational reasons
  - Sparse computation (compared to full deep networks)
  - Shared parameters (only a small number of shared parameters)
  - Translation invariance

Motivation for convolution networks: Gabor functions derived from neuroscience experiments are simple convolutional filters [DL, ch. 9]

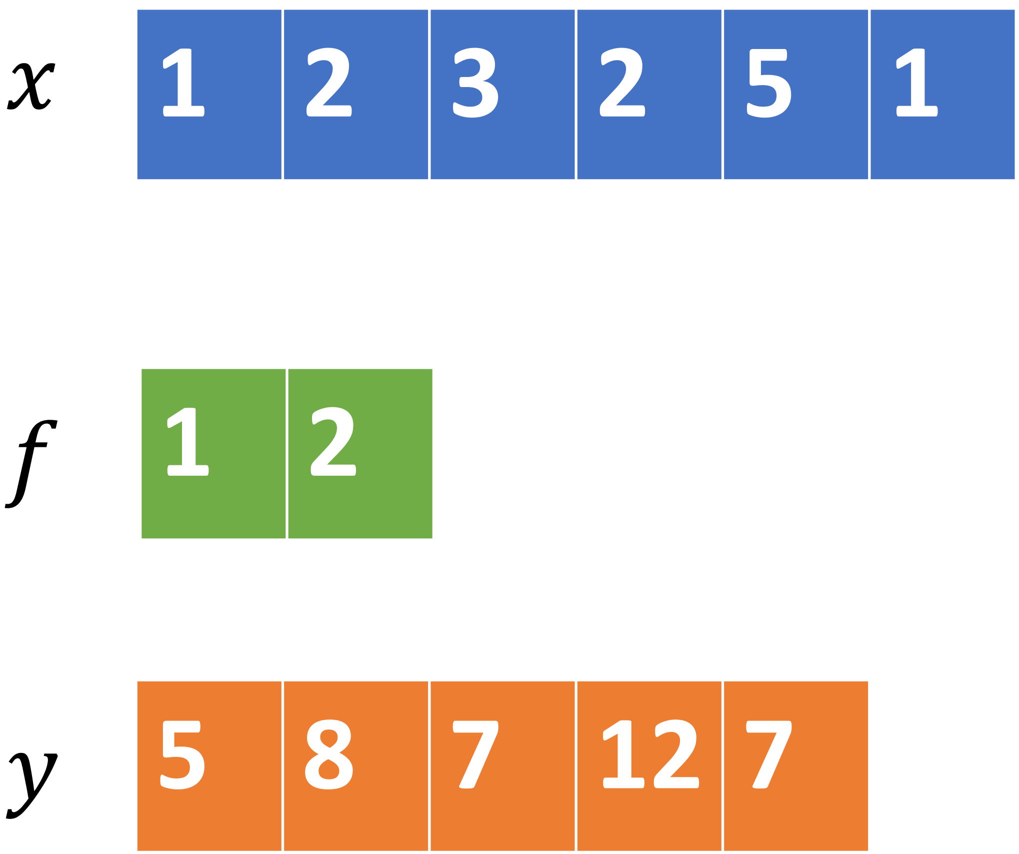


Convolutional networks automatically learn filters similar to Gabor functions [DL, ch. 9]

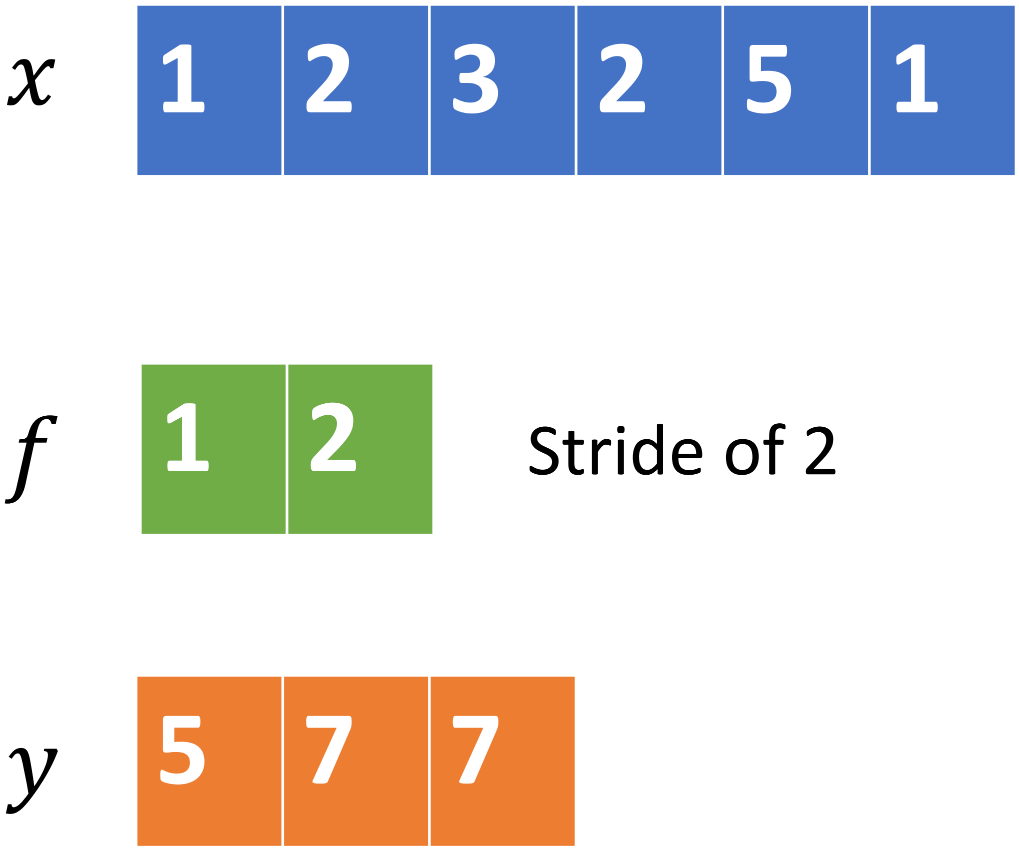




1D convolutions are similar but slightly different than signal processing / math convolutions



Padding or stride parameters alter the computation and output shape



1D convolutions are similar but slightly different than signal processing / math convolutions

x	1	2	3	2	5	1

f	1	2

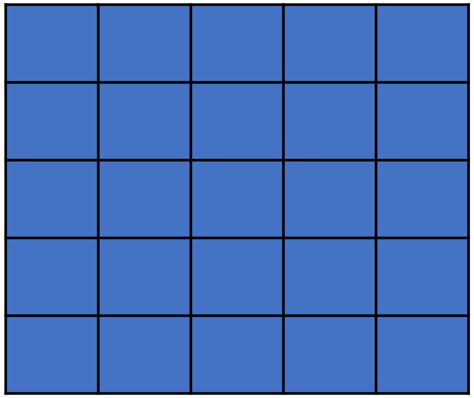
Zero padding of 1

y	2	5	7	8	12	7	1

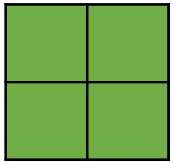
Switch to demo of 1D

2D convolutions are simple generalizations to matrices

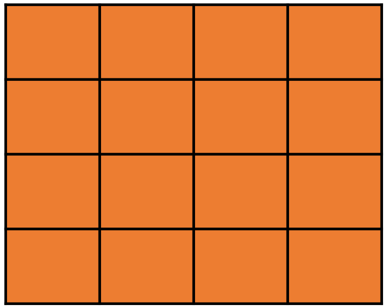
x



f

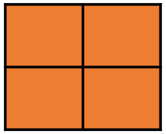


y



Stride of 2

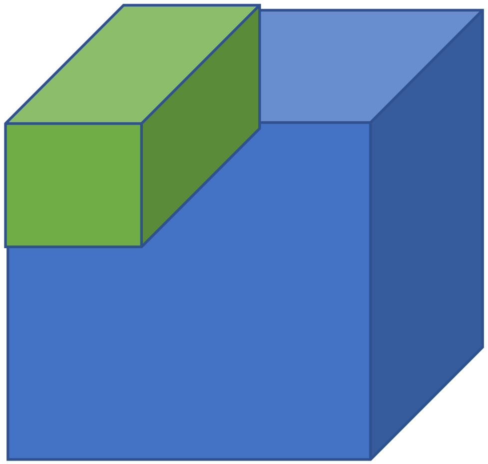
y



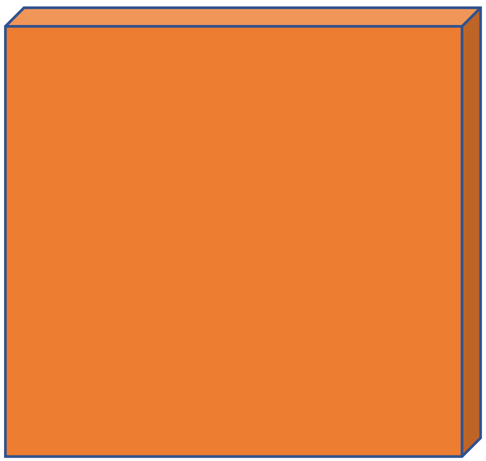
Switch to demo of 2D

3D convolutions are similar but usually channel dimension is assumed

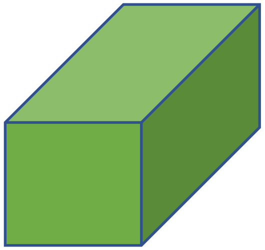
$x \in \mathcal{R}^{c \times w \times h}$



$y \in \mathcal{R}^{1 \times w' \times h'}$

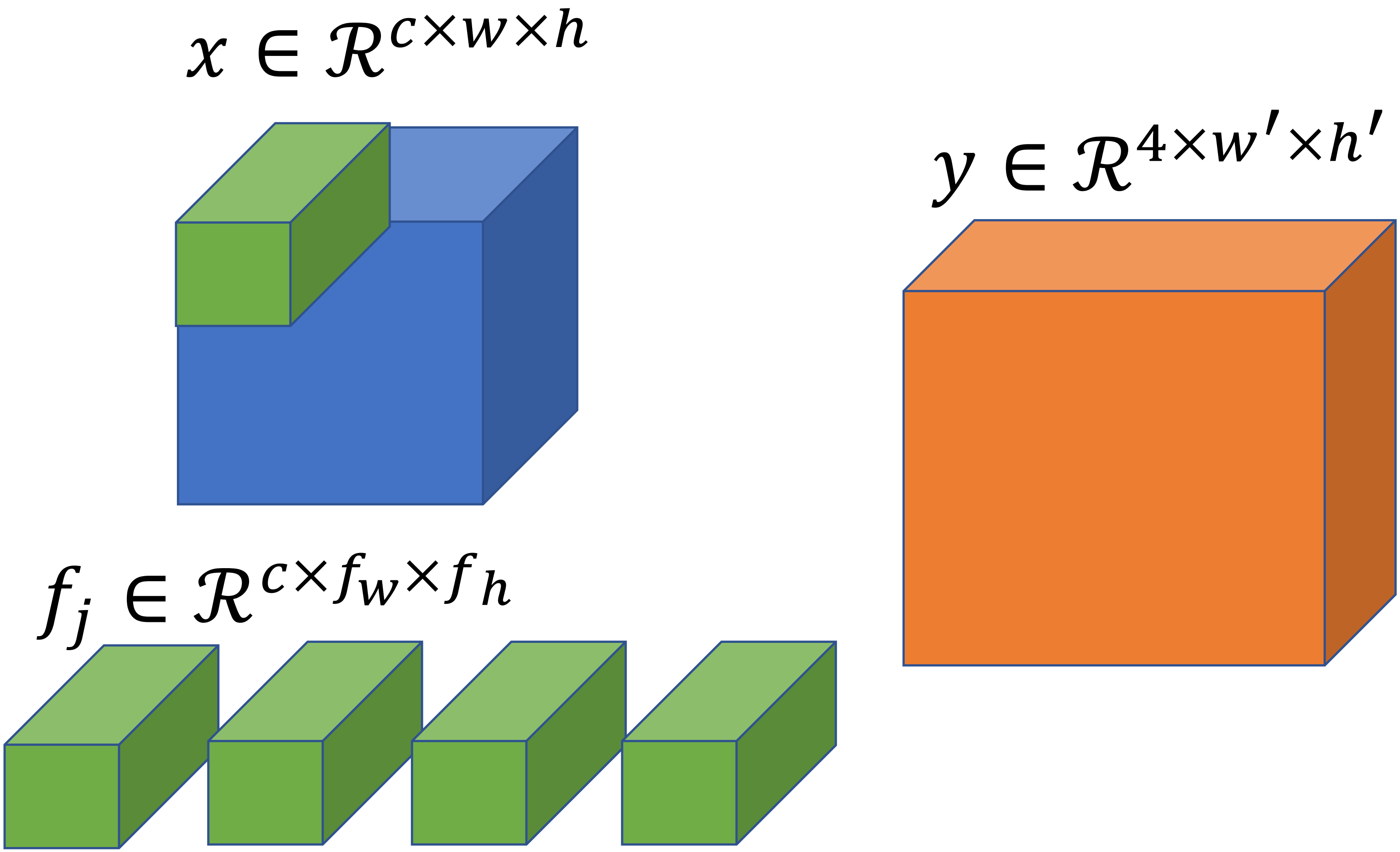


$f \in \mathcal{R}^{c \times f_w \times f_h}$



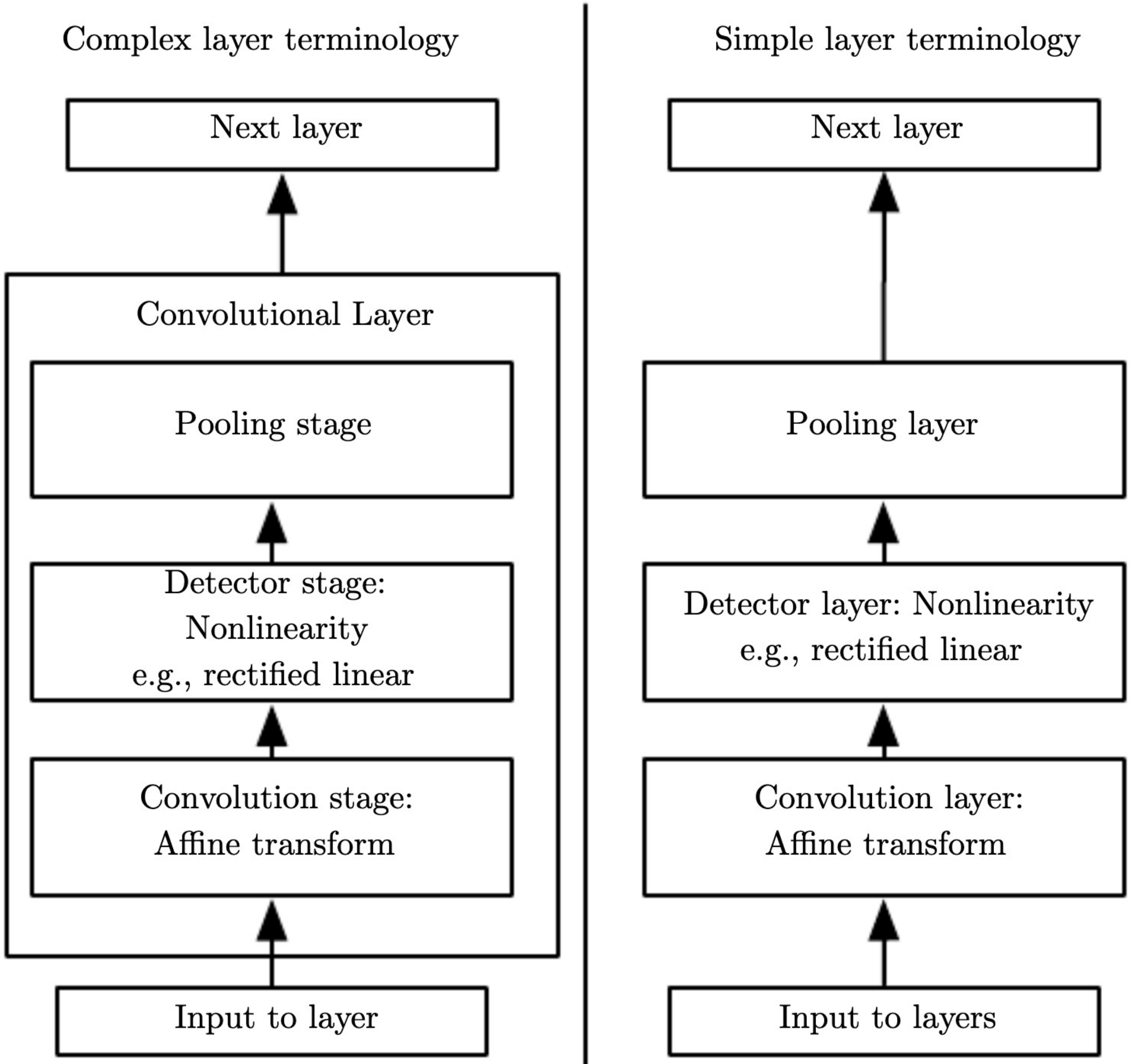
"$f_w \times f_h$ convolution" (channel dimension is assumed)

Multiple convolutions increase the output channel dimension



Switch to demo of 3D

Standard Convolutional Layer Terminology [DL, ch. 9]



Demo of CIFAR-10 CNN in Pytorch

Two important modern CNN architecture concepts: **batch normalization** and **residual networks**

<u>Batch normalization</u> dynamically normalizes each feature to have zero mean and unit variance

- Basic idea: Normalize input batch of each layer **<u>during the forward pass</u>**
  1. Input is **minibatch** of data $X^t \in \mathbb{R}^{m \times d}$ at iteration t
  2. Compute mean and standard deviation for every feature

$$\mu_j^t = \mathbb{E}\left[x_j^t\right], \sigma_j^t = \sqrt{\mathbb{E}\left[\left(x_j^t - \mu_j^t\right)^2\right]}, \qquad \forall j \in \{1, \cdots, d\}$$

  3. Normalize each feature (note **<u>different for every batch</u>**)

$$\tilde{x}_{i,j}^t = \frac{\left(x_{i,j}^t - \mu_j^t\right)}{\sigma_j^t}$$

  4. Output $\tilde{X}^t$

Santurkar, S., Tsipras, D., Ilyas, A., & Madry, A. (2018). How does batch normalization help optimization?. In *Advances in Neural Information Processing Systems* (pp. 2483-2493).

Because BatchNorm removes linear effects, extra linear parameters are also learned

- The form of this final update is:

$$\tilde{x}_{i,j}^t = \frac{(x_{i,j}^t - \mu_j^t)}{\sigma_j^t} \cdot \gamma_j + \beta_j$$

  - Where γ_j and β_j are learnable parameters
  - While μ_j^t and σ_j^t are computed from the **minibatch**
- But how do we compute μ_j^t and σ_j^t about during test time (i.e., no minibatch)?
- Use running average of mean and variance

$$\mu_{run}^t = \lambda \mu_{run}^{t-1} + (1-\lambda)\mu_{batch}^t$$

$${\sigma^2}_{run}^t = \lambda {\sigma^2}_{run}^{t-1} + (1-\lambda){\sigma^2}_{batch}^t$$

For CNNs, the channel dimension is treated as a "feature"

- If the input minibatch tensor is $X^t \in \mathbb{R}^{m \times c \times h \times w}$, then the channel dimension c is treated as a feature:
  $$\mu_j^t = \mathbb{E}[x_j^t], \sigma_j^t = \sqrt{\mathbb{E}\left[\left(x_j^t - \mu_j^t\right)^2\right]},$$
  $$\forall j \in \{1, \cdots, c\}$$
  - Where the mean is taken over **both** the batch dimension m **and** the spatial dimensions h and w
  - Called "Spatial Batch Normalization"
- Variants: Instance, Group or Layer Normalization

https://pytorch.org/docs/stable/nn.html#normalization-layers

<u>BatchNorm</u> can stabilize and accelerate training of deep models

- To use in practice:
  - Only normalize batches during training (`model.train()`)
  - **<u>Turn off</u>** after training (`model.eval()`)
    - Uses running average of mean and variance
- Surprisingly effective at stabilizing training, reducing training time, and producing better models
- Not fully understood why it works

Santurkar, S., Tsipras, D., Ilyas, A., & Madry, A. (2018). How does batch normalization help optimization?. In *Advances in Neural Information Processing Systems* (pp. 2483-2493).

Demo of batch normalization in PyTorch

Residual networks add the input to the output of the CNN

- Most deep model layers have the form:
  $$y = f(x)$$
  - Where f could be any function including a convolutional layer like $f(x) = \sigma\left(\mathrm{Conv}\left(\sigma(\mathrm{Conv}(x))\right)\right)$
- Residual layers add back in the input
  $$y = f(x) + x$$
  - Notice that $f(x)$ models the difference between x and y (hence the name **residual**)

He, K., Zhang, X., Ren, S., & Sun, J. (2016). Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition* (pp. 770-778).

A residual network enables deeper networks because gradient information can flow between layers

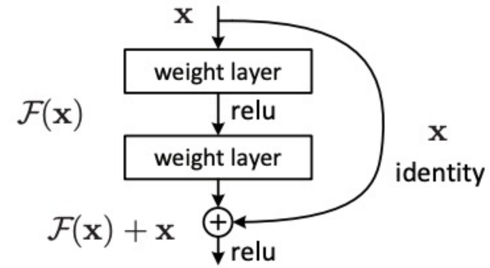
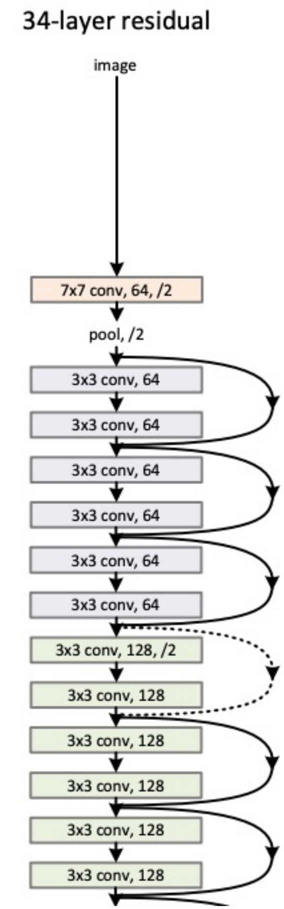


Figure 2. Residual learning: a building block.

- A data flow diagram shows the "shortcut" connections
- Consider composing 2 residual layers:
  - $z^{(1)} = f_1(x) + x$
  - $z^{(2)} = f_2\left(z^{(1)}\right) + z^{(1)}$
  - Or, equivalently
    $z^{(2)} = f_2(f_1(x) + x) + f_1(x) + x$
- If the residuals = 0, then this is merely the identity function



Images from: He, K., Zhang, X., Ren, S., & Sun, J. (2016). Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition* (pp. 770-778).

Detail: If the dimensionality is not the same, then use either fully connected layer or convolution layer to match

- In the 1D case, suppose $f(x): \mathbb{R}^d \to \mathbb{R}^m$, then we need to multiply x by linear operator to match the dimension
$$y = f(x) + Wx, \qquad \text{where } W \in \mathbb{R}^{m \times d}$$

- Similarly, for images, if $f(x): \mathbb{R}^{c \times h \times w} \to \mathbb{R}^{c' \times h' \times w'}$, we can apply a convolution layer to match the dimensions
$$y = f(x) + \mathrm{conv}(x),$$
$$\text{where } \mathrm{conv}(\cdot): \mathbb{R}^{c \times h \times w} \to \mathbb{R}^{c' \times h' \times w'}$$

Demo of CNN with very simple residual network