(from slides) Why convolutions?

Neuroscientific inspiration

Computational reasons

1. Sparse computation (compared to full deep networks)
2. Shared parameters (only a small number of shared parameters)
3. Translation invariance

1D convolutions, similar but slightly different than signal processing / math convolutions

[-1, 1] filter/kernel highlights "sharp points" of signal

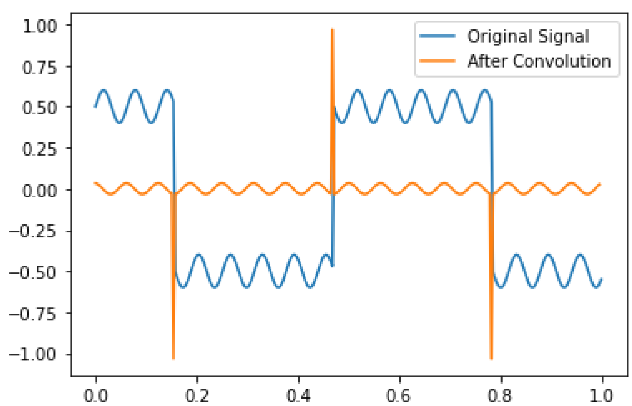
In [1]:

```
import torch
import matplotlib.pyplot as plt
%matplotlib inline

t = torch.linspace(0, 1.0, 300)
x = (torch.cos(10*t) > 0.0).float() + 0.1*torch.sin(100*t)-0.5
plt.plot(t.numpy(), x.numpy(), label='Original Signal')

from torch.nn import functional as F
filt = torch.tensor([-1, 1.0])
print('Filter')
print(filt)
# Should have shape $(m, c, w)$ where m is minibatch size, c is # channels and w is width
y = F.conv1d(x.reshape(1, 1, len(x)), filt.reshape(1, 1, len(filt))).squeeze_()
plt.plot(t.numpy()[:len(y)], y.numpy(), label='After Convolution')
plt.legend()
```

```
Filter
tensor([-1.,  1.])
```

Out[1]:

```
<matplotlib.legend.Legend at 0x7fb7d387ff70>
```



Convolutions are linear operators (i.e., matrix multiplication) with shared parameters

In [2]:

```
x = torch.randn(10).float().requires_grad_(True)
filt = torch.tensor([-1, 1]).float()
#filt = torch.tensor([1, 2, 3, 4]).float()
y = F.conv1d(x.reshape(1, 1, len(x)), filt.reshape(1, 1, len(filt))).squeeze_()
```

```
def extract_jacobian(x, y):
    J = torch.zeros((len(y), len(x))).float()
    for i in range(len(y)):
        v = torch.zeros(len(y)).float()
        v[i] = 1
        if x.grad is not None:
            x.grad.zero_()
        y.backward(v, retain_graph=True)
        J[i, :] = x.grad
    return J

A = extract_jacobian(x, y)
print(A)
y2 = torch.matmul(A, x)
print(y)
print(y2)
print(y-y2)
```

```
tensor([[-1.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
        [ 0., -1.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.],
        [ 0.,  0., -1.,  1.,  0.,  0.,  0.,  0.,  0.,  0.],
        [ 0.,  0.,  0., -1.,  1.,  0.,  0.,  0.,  0.,  0.],
        [ 0.,  0.,  0.,  0., -1.,  1.,  0.,  0.,  0.,  0.],
        [ 0.,  0.,  0.,  0.,  0., -1.,  1.,  0.,  0.,  0.],
        [ 0.,  0.,  0.,  0.,  0.,  0., -1.,  1.,  0.,  0.],
        [ 0.,  0.,  0.,  0.,  0.,  0.,  0., -1.,  1.,  0.],
        [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0., -1.,  1.]])
tensor([ 2.1247, -0.5649, -0.7516, -0.5571, -1.3838,  2.7994, -0.0724, -1.6590,
        -0.0845], grad_fn=<SqueezeBackward2>)
tensor([ 2.1247, -0.5649, -0.7516, -0.5571, -1.3838,  2.7994, -0.0724, -1.6590,
        -0.0845], grad_fn=<MvBackward>)
tensor([0., 0., 0., 0., 0., 0., 0., 0., 0.], grad_fn=<SubBackward0>)
```

2D convolutions are similar and can be applied to images

Different filters extract different features from the image

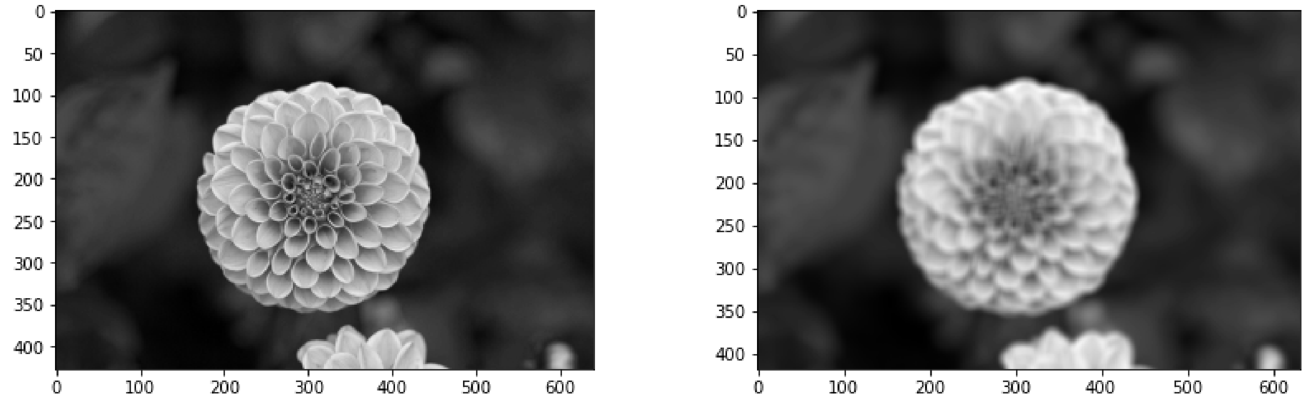
In [3]:

```
import sklearn.datasets
A = torch.tensor(sklearn.datasets.load_sample_image('china.jpg')).float()
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float()
A = torch.sum(A, dim=2) # Sum channels

filt = torch.tensor([[-1, 0, 1], [-1, 0, 1], [-1, 0, 1]]).float() # Horizontal
filt = torch.tensor([[-1, 0, 1], [-1, 0, 1], [-1, 0, 1]]).float().t() # Vertical
filt = torch.tensor([[1, -1], [-1, 1]]).float() # Checker board pattern
filt = torch.ones((10, 10)).float() # Blur
print('Filter')
print(filt)
B = F.conv2d(A.reshape(1, 1, *A.size()), filt.reshape(1, 1, *filt.size()), padding=1).squeeze()
B = F.conv2d(A.reshape(1, 1, *A.size()), filt.reshape(1, 1, *filt.size())).squeeze()
print('A size', A.size(), 'B size', B.size())

fig, axes = plt.subplots(1, 2, figsize=(14,4))
axes[0].imshow(A.numpy(), cmap='gray')
axes[1].imshow(B.numpy(), cmap='gray')
```

```
Filter
tensor([[1., 1., 1., 1., 1., 1., 1., 1., 1., 1.],
        [1., 1., 1., 1., 1., 1., 1., 1., 1., 1.],
        [1., 1., 1., 1., 1., 1., 1., 1., 1., 1.],
        [1., 1., 1., 1., 1., 1., 1., 1., 1., 1.],
        [1., 1., 1., 1., 1., 1., 1., 1., 1., 1.],
        [1., 1., 1., 1., 1., 1., 1., 1., 1., 1.],
        [1., 1., 1., 1., 1., 1., 1., 1., 1., 1.],
        [1., 1., 1., 1., 1., 1., 1., 1., 1., 1.],
        [1., 1., 1., 1., 1., 1., 1., 1., 1., 1.],
        [1., 1., 1., 1., 1., 1., 1., 1., 1., 1.]])
A size torch.Size([427, 640]) B size torch.Size([418, 631])
```

Out[3]:

```
<matplotlib.image.AxesImage at 0x7fb7d88f8370>
```



Higher dimensional convolutions are similar (i.e., if there is more than 1 channel)

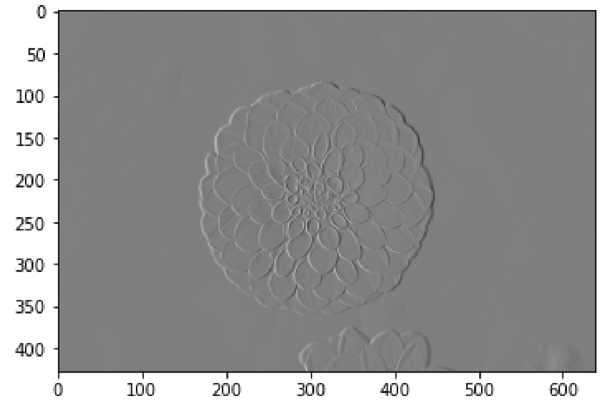
In [4]:

```
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float()
A = A/255
A = A.permute(2,0,1)
print(A.size())

filt = torch.tensor([1, 0, 0]).reshape(3, 1, 1).float() # Extract red
filt = torch.tensor([0, 1, 0]).reshape(3, 1, 1).float() # Extract green
filt = torch.tensor([0, 0, 1]).reshape(3, 1, 1).float() # Extract blue
filt = torch.ones(3, 5, 5).float() # Blur
filt = torch.tensor([
    [[-1, 1]],
    [[-1, 1]],
    [[-1, 1]],
]).float()

print('Filter')
print(filt)
print(filt.size())
B = F.conv2d(A.reshape(1, *A.size()), filt.reshape(1, *filt.size())).squeeze()
print('A size', A.size(), 'B size', B.size())

fig, axes = plt.subplots(1, 2, figsize=(14,4))
axes[0].imshow(A.permute(1,2,0), cmap='gray')
axes[1].imshow(B, cmap='gray')
```

```
torch.Size([3, 427, 640])
Filter
tensor([[[[-1.,  1.]],

        [[-1.,  1.]],

        [[-1.,  1.]]]])
torch.Size([3, 1, 2])
A size torch.Size([3, 427, 640]) B size torch.Size([427, 639])
```

Out[4]:

```
<matplotlib.image.AxesImage at 0x7fb7da0b8dc0>
```



How to interpret convolution descriptions (usually)

Kernel sizes assume all channels (e.g., "1x1 convolution" corresponds to a kernel size of 1x1xC where C is the number of channles)

The number of filters in the previous layer corresponds to the number of channels in the curent layer

Why convolutions again?

Computational reasons

1. Sparse computation (compared to full deep networks)
2. Shared parameters (only a small number of shared parameters)
3. Translation invariance

Extract image features (edges, etc.)

Automatically learn image features

Need several other components for extracting features: Activation functions and pooling layers

Why activation functions? Activation functions enable non-linear models

Consider a deep linear network

In [5]:
```
torch.manual_seed(0)
A1 = torch.randn((10, 5))
A2 = torch.randn((10, 10))
A3 = torch.randn((1, 10))

x = torch.randn(5)
print('x', x)
y = torch.matmul(A1, x)
y = torch.matmul(A2, y)
y = torch.matmul(A3, y)
```

```
print('y', y)

b = torch.matmul(A3, torch.matmul(A2, A1))
y2 = torch.matmul(b, x)
print('y2', y2)
```

```
x tensor([ 1.4875, -0.2230, -1.0057, -0.4139,  1.1600])
y tensor([4.1752])
y2 tensor([4.1752])
```

If you add activation functions, the deep function cannot be simplified

In [6]:

```
torch.manual_seed(0)
A1 = torch.randn((10, 5))
A2 = torch.randn((10, 10))
A3 = torch.randn((1, 10))

x = torch.randn(5)
print('x', x)
y = torch.matmul(A1, x)
y = torch.relu(y)
y = torch.matmul(A2, y)
y = torch.relu(y)
y = torch.matmul(A3, y)
print('y', y)

b = torch.matmul(A3, torch.matmul(A2, A1))
y2 = torch.matmul(b, x)
print('y2', y2)
```

```
x tensor([ 1.4875, -0.2230, -1.0057, -0.4139,  1.1600])
y tensor([18.9449])
y2 tensor([4.1752])
```

Without ReLU or activation function, the function can only be linear

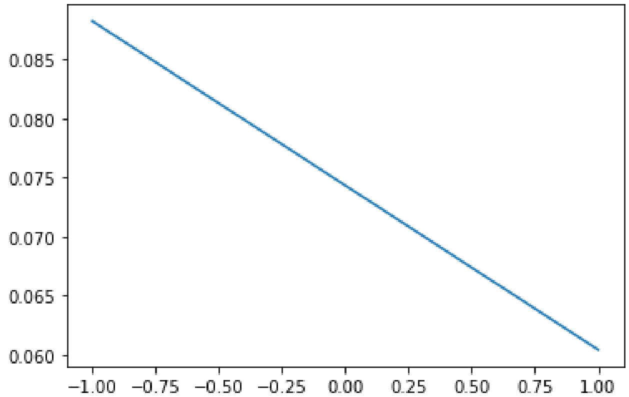
In [7]:

```
N, D_in, H, D_out = 64, 1, 10, 1
model = torch.nn.Sequential(
    torch.nn.Linear(D_in, H),
    torch.nn.Linear(H, D_out),
)
x = torch.linspace(-1, 1).reshape(-1, 1)
y = model(x)
plt.plot(x.detach().numpy(), y.detach().numpy())
```

```
<ipython-input-7-6cbdf43a82fe>:6: UserWarning: Not providing a value for linspace's steps is depreca
ted and will throw a runtime error in a future release. This warning will appear only once per proce
ss. (Triggered internally at  /Users/distiller/project/conda/conda-bld/pytorch_1623459044803/work/at
en/src/ATen/native/RangeFactories.cpp:25.)
  x = torch.linspace(-1, 1).reshape(-1, 1)
```

Out[7]:

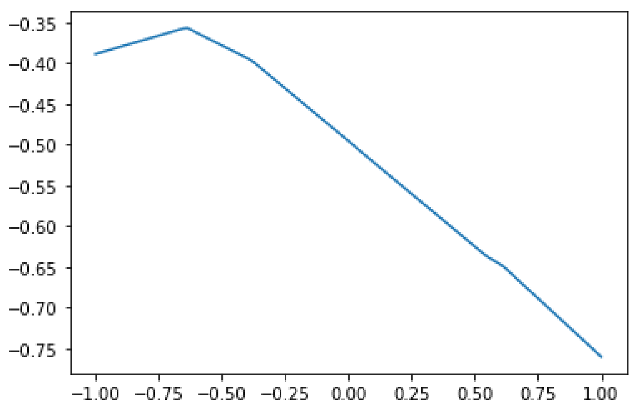
```
[<matplotlib.lines.Line2D at 0x7fb7da05b700>]
```



With ReLU activation function, the function is *piecewise* linear

In [8]:
```
N, D_in, H, D_out = 64, 1, 10, 1
model = torch.nn.Sequential(
    torch.nn.Linear(D_in, H),
    torch.nn.ReLU(),
    torch.nn.Linear(H, D_out),
)
x = torch.linspace(-1, 1).reshape(-1, 1)
y = model(x)
plt.plot(x.detach().numpy(), y.detach().numpy())
```

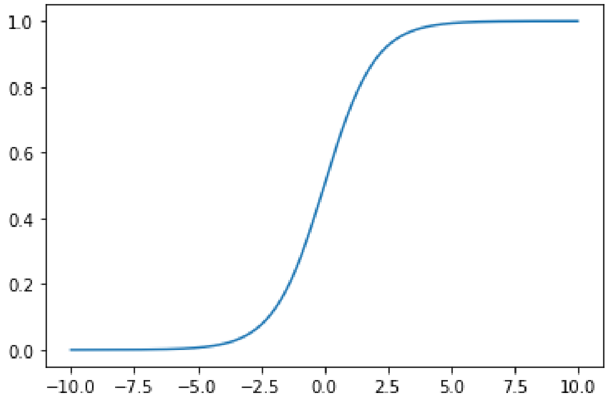
Out[8]: [<matplotlib.lines.Line2D at 0x7fb7da959be0>]



Common activation functions include sigmoid, ReLU, Leaky ReLU, tanh

In [9]:
```
t = torch.linspace(-10, 10, 300)
plt.plot(t.numpy(), torch.sigmoid(t).numpy())
```

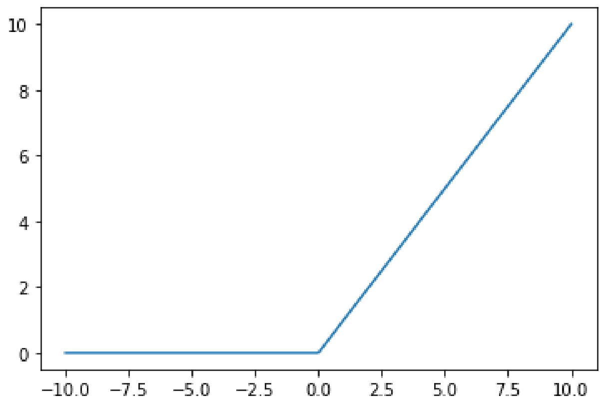
Out[9]: [<matplotlib.lines.Line2D at 0x7fb7d92bf850>]



In [10]:

```
plt.plot(t.numpy(), F.relu(t).numpy())
```

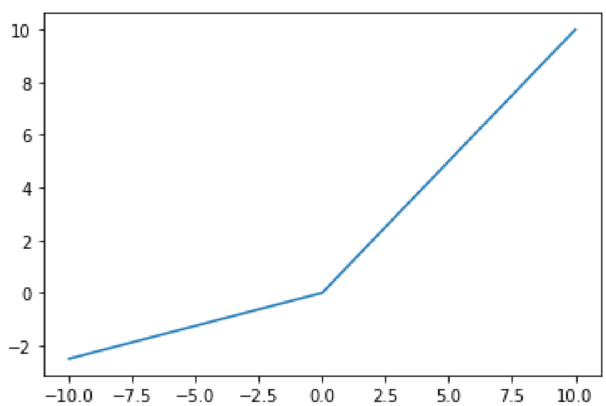
Out[10]: [<matplotlib.lines.Line2D at 0x7fb7d9df4790>]



In [11]:

```
plt.plot(t.numpy(), F.leaky_relu(t, negative_slope=0.25).numpy())
```

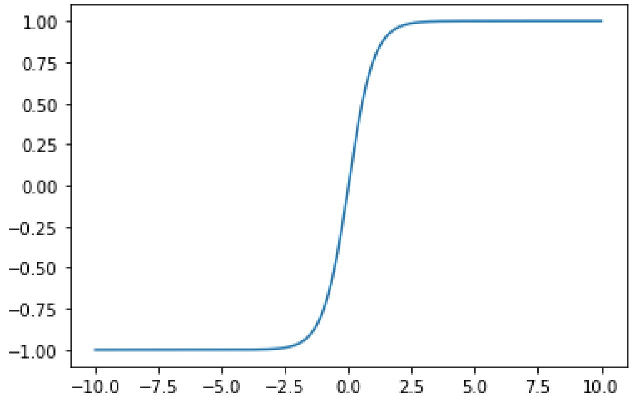
Out[11]: [<matplotlib.lines.Line2D at 0x7fb7d9703fd0>]



In [12]:

```
plt.plot(t.numpy(), torch.tanh(t).numpy())
```

Out[12]: [<matplotlib.lines.Line2D at 0x7fb7d97dad90>]



Pooling layers are used to reduce dimensionality and introduce some location invariance

Pooling layers include max pooling and average pooling

In [13]:

```
torch.manual_seed(0)
x = torch.randint(10, (10,)).float()
y = F.max_pool1d(x.reshape(1,1,-1), kernel_size=3)
y2 = F.max_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1)
y3 = F.max_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1, padding=1)
y = F.avg_pool1d(x.reshape(1,1,-1), kernel_size=3)
y2 = F.avg_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1)
y3 = F.avg_pool1d(x.reshape(1,1,-1), kernel_size=3, stride=1, padding=1)
print(x)
print(y)
print(y2)
print(y3)
```

```
tensor([4., 9., 3., 0., 3., 9., 7., 3., 7., 3.])
tensor([[[5.3333, 4.0000, 5.6667]]])
tensor([[[5.3333, 4.0000, 2.0000, 4.0000, 6.3333, 6.3333, 5.6667, 4.3333]]])
tensor([[[4.3333, 5.3333, 4.0000, 2.0000, 4.0000, 6.3333, 6.3333, 5.6667,
          4.3333, 3.3333]]])
```

In [17]:

```
x = torch.randn((3,4,10,20)).float()
print(x.shape, 'N x C x H x W')
y = F.max_pool2d(x, kernel_size=2)
print(y.shape, 'The number of channels does not change for pooling')
y2 = F.max_pool2d(x, kernel_size=2)
print(y2.shape, 'Note that `stride=kernel_size` by default')
y3 = F.max_pool2d(x, kernel_size=2, stride=1)
print(y3.shape, 'Can set stride explicitly to 1')
y4 = F.max_pool2d(x, kernel_size=3, stride=1, padding=1)
print(y4.shape, 'Can produce the same size')
```

```
torch.Size([3, 4, 10, 20]) N x C x H x W
torch.Size([3, 4, 5, 10]) The number of channels does not change for pooling
torch.Size([3, 4, 5, 10]) Note that `stride=kernel_size` by default
torch.Size([3, 4, 9, 19]) Can set stride explicitly to 1
torch.Size([3, 4, 10, 20]) Can produce the same size
```

Is average pooling a linear or non-linear operation?

Is max pooling a linear or non-linear operation?

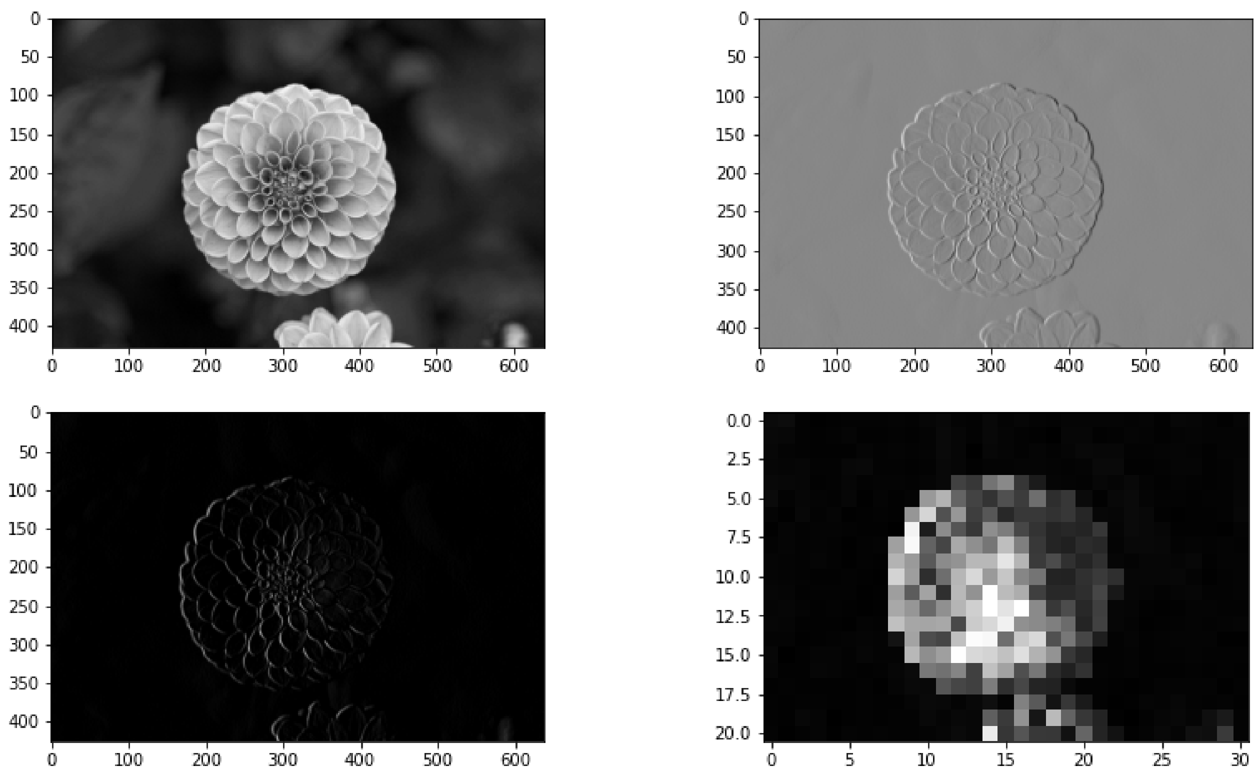
Convolution Neural Network (CNN) layers are compositions of convolution, activation and pooling

(See illustration on slide)

In [15]:

```
import sklearn.datasets
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float()
A = torch.sum(A, dim=2)
filt = torch.tensor([[-1, 0, 1], [-1, 0, 1], [-1, 0, 1]]).float() # Horizontal
#filt = torch.tensor([[-1, 0, 1], [-1, 0, 1], [-1, 0, 1]]).float().t() # Vertical
#filt = torch.tensor([[1, -1], [-1, 1]]).float() # Checker board pattern
#filt = torch.ones((10, 10)).float() # Blur
print('Filter')
print(filt)
B = F.conv2d(A.reshape(1, 1, *A.size()), filt.reshape(1, 1, *filt.size()))
print('A size', A.size(), 'B size', B.size())
C = torch.relu(B)
D = torch.max_pool2d(C, kernel_size=20)
#D = torch.max_pool2d(C, kernel_size=20, stride=1)

fig, axes = plt.subplots(2, 2, figsize=(14,8))
axes = axes.ravel()
for im, ax in zip([A, B, C, D], axes):
    ax.imshow(im.squeeze(), cmap='gray')
```

```
Filter
tensor([[-1.,  0.,  1.],
        [-1.,  0.,  1.],
        [-1.,  0.,  1.]])
A size torch.Size([427, 640]) B size torch.Size([1, 1, 425, 638])
```



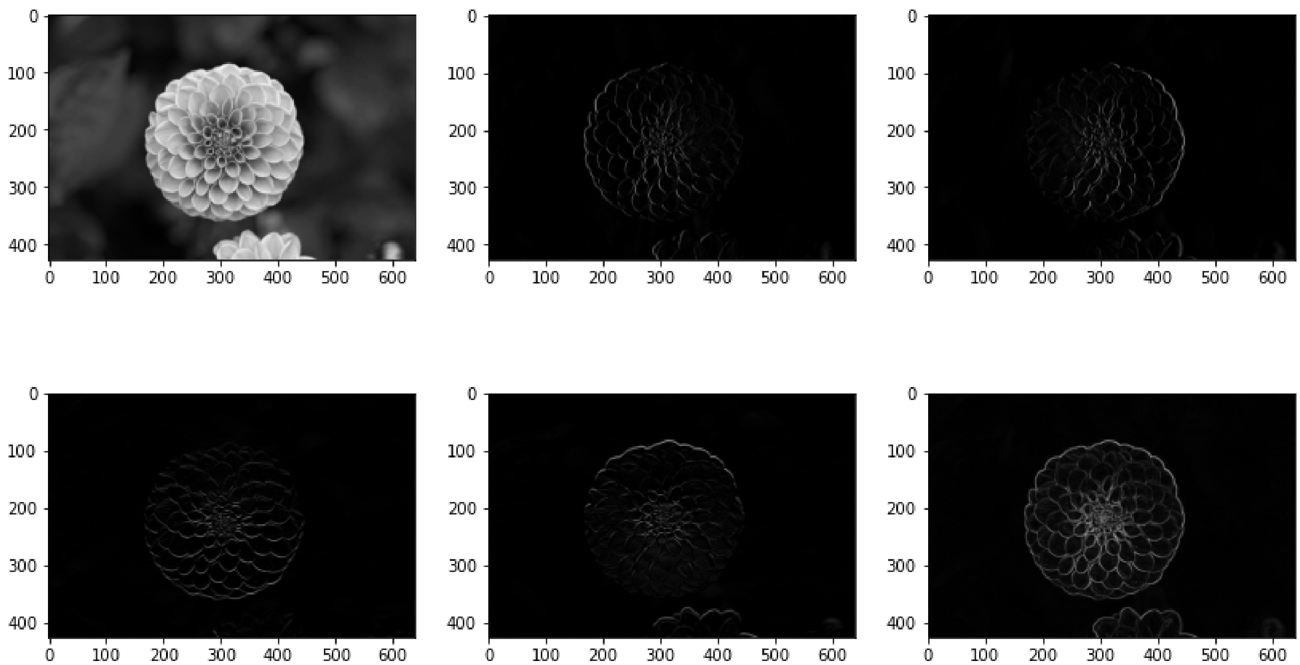
How could you detect an edge from multiple angles by combining convolutions and ReLUs?

Hint: First detect edges from all directions, then combine.

In [16]:

```
import sklearn.datasets
import torch
import numpy as np
A = torch.tensor(sklearn.datasets.load_sample_image('china.jpg')).float()
A = torch.tensor(sklearn.datasets.load_sample_image('flower.jpg')).float()
```

```
A = torch.sum(A, dim=2)

filters = torch.tensor([
    [[[-1, 1], [-1, 1]]],
    [[[1, -1], [1, -1]]],
    [[[1, 1], [-1, -1]]],
    [[[-1, -1], [1, 1]]],
]).float()
B = F.conv2d(A.reshape(1, 1, *A.size()), filters)
C = torch.relu(B)

# Combine
filt = torch.ones(4).float()
D = F.conv2d(C, filt.reshape(1, 4, 1, 1))

fig, axes = plt.subplots(2, 3, figsize=(14,8))
for im, ax in zip([A, *C[0,:,:,:], D], axes.ravel()):
    ax.imshow(im.squeeze(), cmap='gray')
```



Check out PyTorch tutorial on simple classifier on CIFAR10 dataset:

https://pytorch.org/tutorials/beginner/blitz/cifar10_tutorial.html