

## Quiz 0 (prerequisite quiz, ungraded)

STUDENT NAME

### Q1 Instructions

0 Points

This prerequisite quiz will NOT be taken for a grade. This is just to help you know if you are ready for this class or should consider taking it at a later time. I give a few general guidelines below though each person is different.

If you are comfortable solving 90% or more of these questions, then will probably do fine in this class.

If you are comfortable with about 80% of these questions, you will probably need to learn several background concepts independently on your own as you go through the course and the project. The class is geared towards first-year graduate students and independence in learning new concepts and skills is expected.

If you are comfortable with less than 70% of these questions, you may want to consider taking the class at a different time.

Note: Regular quizzes will not be this long. When giving numbers as short answers, please give in standard decimal notation with preceding 0 if needed but no trailing 0s (e.g., "0.15", "2.9", "100" but NOT "0.15000" NOR ".15" NOR "6.0").

### Q2 Probability and random variables

6 Points

#### Q2.1

1 Point

Suppose  $X$  is a discrete random variable with the following probability mass function (PMF):  $P(X = 1) = 0.2, P(X = 2) = 0.5, P(X = 3) = 0.3$ .

What is the mean of this random variable (i.e.,  $\mathbb{E}[X]$ , where  $\mathbb{E}$  denotes expectation)?

Save Answer

### Q2.2

1 Point

For the same random variable  $X$  above, what is  $\mathbb{E}[X^2 + X - 1]$ ?

Enter your answer here

Save Answer

### Q2.3

1 Point

Let the random variables  $X$  and  $Y$  have the following joint probability distribution:

$$\begin{aligned} P(X = 0, Y = 0) &= 0.1 & P(X = 1, Y = 0) &= 0.3 \\ P(X = 0, Y = 1) &= 0.4 & P(X = 1, Y = 1) &= 0.2 \end{aligned}$$

What is the marginal probability of  $X = 0$  (i.e., what is  $P(X = 0)$ )?

Enter your answer here

Save Answer

### Q2.4

1 Point

For the same random variables  $X$  and  $Y$  above.

What is the conditional probability of  $Y = 1$  given that we know that  $X = 1$  (i.e., what is  $P(Y = 1|X = 1)$ )?

Enter your answer here

Save Answer

### Q2.5

1 Point

Suppose  $X$  is a continuous random variable that has the following probability density function (PDF):

$$f(x) = \begin{cases} 1.5, & \text{if } 0 \leq x < 0.5 \\ 0.5, & \text{if } 0.5 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

What is the cumulative distribution function (CDF) at  $X = 0.5$  (i.e.,  $F(0.5)$ ) where  $F$  is the CDF of  $X$ ? (Remember that a CDF is defined as  $F(x) = \int_{-\infty}^x f(z)dz$ .)

### Q2.6

1 Point

(Bonus, harder) Suppose the random variable  $U$  is distributed as a uniform distribution between 0 and 1, and let  $F_X$  and  $F_X^{-1}$  be the CDF and inverse CDF (or quantile) functions of an exponential random variable  $X$  with  $\lambda = 2$  (see [https://en.wikipedia.org/wiki/Exponential\\_distribution](https://en.wikipedia.org/wiki/Exponential_distribution)).

What is the PDF of the random variable  $Z$  defined as  $Z \triangleq F_X^{-1}(U)$ ?

### Q3 Linear algebra

5 Points

#### Q3.1

1 Point

Write the following linear equation in terms of the column vectors  $\beta = [\beta_1, \beta_2, \beta_3]^T$  and  $\mathbf{x} = [x_1, x_2, x_3]^T$ , and the scalar  $y$ :

$$y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 .$$

#### Q3.2

1 Point

Suppose you have the following quadratic equation:

$$y = a_{11}x_1^2 + a_{12}x_1x_2 + a_{21}x_2x_1 + a_{22}x_2^2$$

Write this equation in terms of the matrix  $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ , the column vector  $\mathbf{x} = [x_1, x_2]^T$ ,

and the scalar  $y$ :

### Q3.3

1 Point

Let  $\mathbf{x} = [1, 2, 5, 10]^T$  be a 4D column vector. What is magnitude of this vector when projected along the linear direction defined by the vector  $\mathbf{v} = [0.6, 0, 0, 0.8]^T$ ?

### Q3.4

1 Point

Let  $Q$  be a  $d \times d$  orthogonal matrix and let  $\mathbf{x}$  be a  $d$ -dimensional column vector.

If we define the function  $g$  to be  $g(\mathbf{x}) = Q\mathbf{x}$ , what is the inverse of  $g$  (i.e., what is  $g^{-1}(\mathbf{x})$ )?

### Q3.5

1 Point

What is the  $\ell_2$  norm of the vector  $\mathbf{x} = [3, 4]$  (i.e., what is  $\|\mathbf{x}\|_2$ )?

## Q4 Programming

4 Points

#### Q4.1

1 Point

The following function is an attempt to compute the average value and averaged squared value of a list of numbers. However, there are two bugs, can you find the two bugs and correct the code?

```
def avg_and_squared_avg(x):  
    sum = 0  
    for i in range(len(x)-1):  
        sum = sum + x[i]  
    avg = sum/len(x)  
    for a in x:  
        sum += a*a  
    squared_avg = sum/len(x)  
    return avg, squared_avg
```

For example, if we call this function as follows:

`avg_and_squared_avg([1, 2, 3, 4])` the output is `(1.5, 9.0)` but the expected output is `(2.5, 7.5)`

Enter your answer here

Save Answer

#### Q4.2

1 Point

Write a function that takes a positive integer as input and returns a boolean indicating whether the number is prime or not. Do not worry much about efficiency. [Hint: Check if the number is divisible by any integer smaller than it but greater than 1.]

Enter your answer here

Save Answer

#### Q4.3

1 Point

Write a function that takes a list of integers as input and returns a list of the numbers in the list that are divisible by 2 and 3. For example, `get_divisible_by_2_and_3([2,6,12,5,60])` would return `[6,12,60]`.

Enter your answer here

Save Answer

#### Q4.4

1 Point

(Bonus, harder) Write a histogram function (see <https://en.wikipedia.org/wiki/Histogram>) in Python where the first argument is the data as a list of numbers and the second argument is the number of bins. The minimum and maximum values are assumed to be 0 and 1 (i.e., 0 and 1 should be the first and last bin edge). You can assume data only contains numbers between 0 and 1.

The output should be a list of counts for each bin (i.e., the number of data points that fall into each bin) and the bin edges. Do not worry about efficiency at this point.

For example, `histogram([0.1, 0.55, 0.85, 0.15], 5)`, the output should be `([2, 0, 1, 0, 1], [0, 0.2, 0.4, 0.6, 0.8, 1.0])`.

Enter your answer here

Save Answer

Save All Answers

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