Reinforcement Learning: Practical Algorithms

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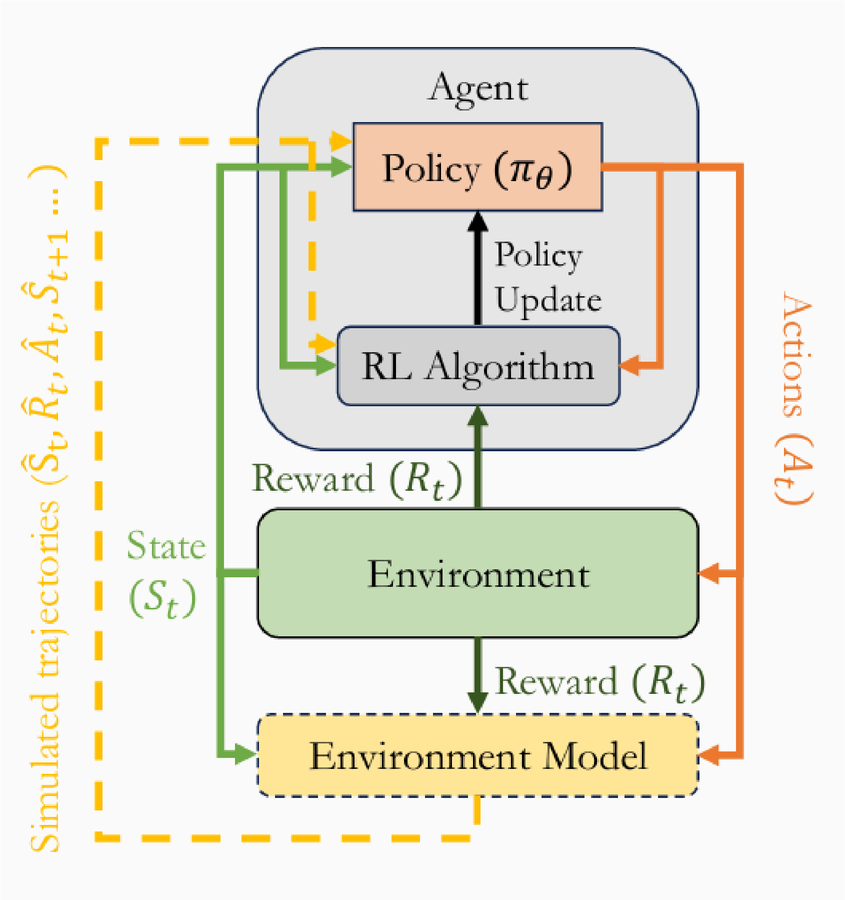
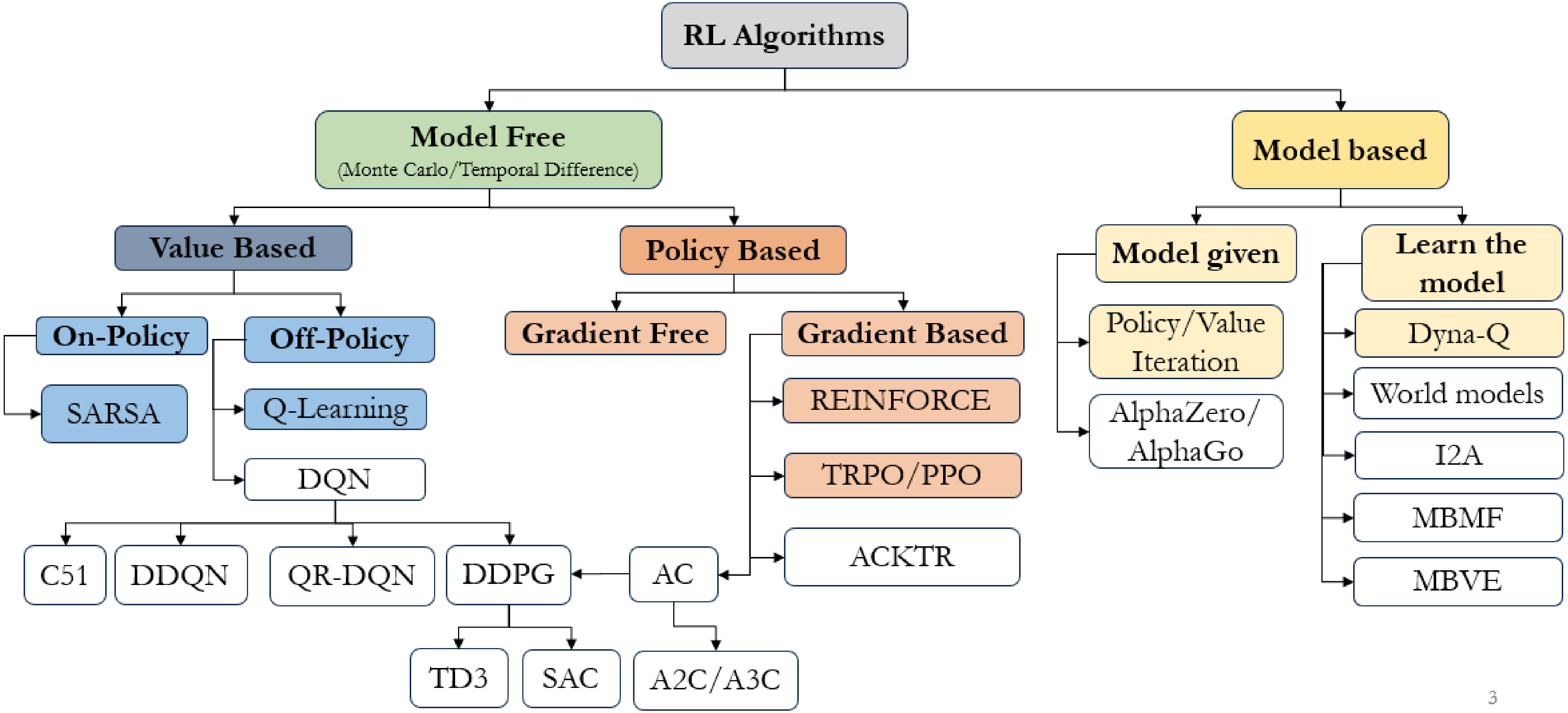
Reinforcement Learning Algorithms Overview

- Recall that our aim is to find the optimal policy which maximizes the expected return (discounted sum of future rewards).
- Policies can be compared based on value functions (policy ≈ value function), thus need a way to compute value function (**Prediction**) – **Policy Evaluation**.
- Starting with an arbitrary policy improve the policy to reach optimal policy (**Control**) – **Policy Iteration**.
  - Optimal policy can be constructed from optimal value function, improve value function – **Value Iteration**.
- What if environment(MDP) is unknown?
  - **Estimate** value function via. reward sampling (**Model Free**).
  - Or learn a model of the environment (**Model Based**), then compute value function (simulated experience).
- What if MDP has continuous or infinite states?
  - Use **parameterized function approximators** for value function (**Value based**) or policy (**Policy Based**).
  - Search or learn parameters (**gradient free** or **gradient based** searching).



Categorizing RL Algorithms



RL Algorithms

Model Free
(Monte Carlo/Temporal Difference)

Model based

Value Based

Policy Based

Model given

Learn the model

On-Policy

Off-Policy

Gradient Free

Gradient Based

SARSA

Q-Learning

DQN

C51

DDQN

QR-DQN

DDPG

TD3

SAC

AC

A2C/A3C

REINFORCE

TRPO/PPO

ACKTR

Policy/Value Iteration

AlphaZero/ AlphaGo

Dyna-Q

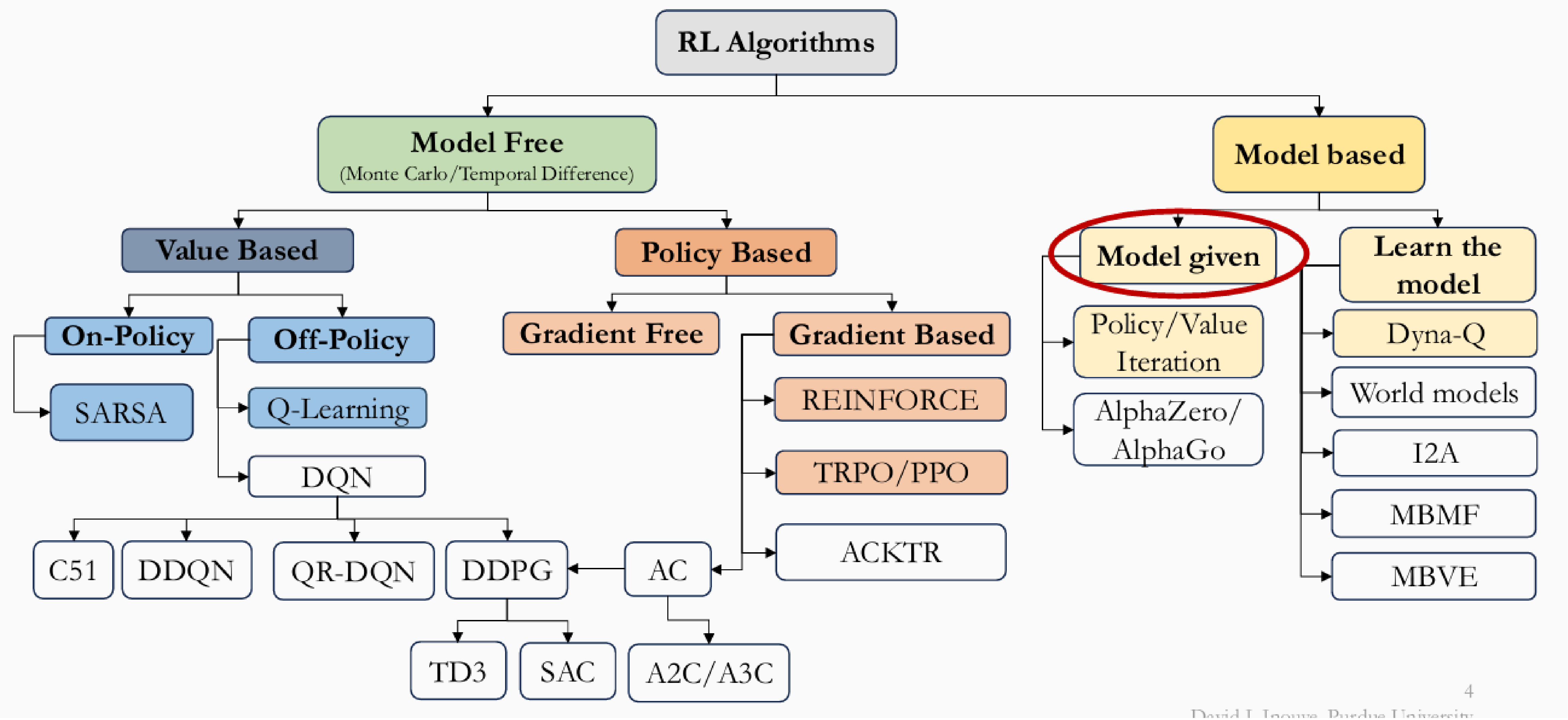
World models

I2A

MBMF

MBVE

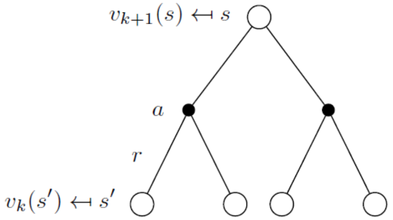
Categorizing RL Algorithms



(1.A) Policy **Evaluation** – How Good Is Your Policy?

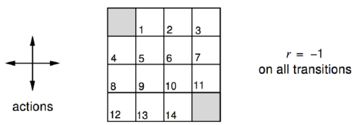
- Evaluate a given policy π, estimate v_π. Also known as a **Prediction** problem.
- **Input:** Known MDP $(\mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma)$ and policy π.
- **Output:** Value function v_π.
- **Solution:** Iterative application of **Bellman equation** and dynamic programming.
- At each iteration $k+1$, update $v_{k+1}(s)$ from $v_k(s')$:

$$v_{k+1}(s) = \sum_{a,r,s'} \pi(a \mid s)\, p(s', r \mid s, a)\, [r + \gamma\, v_k(s')]$$



$$v_{k+1}(s) = \sum_{a \in \mathcal{A}} \pi(a|s) \left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_k(s') \right)$$

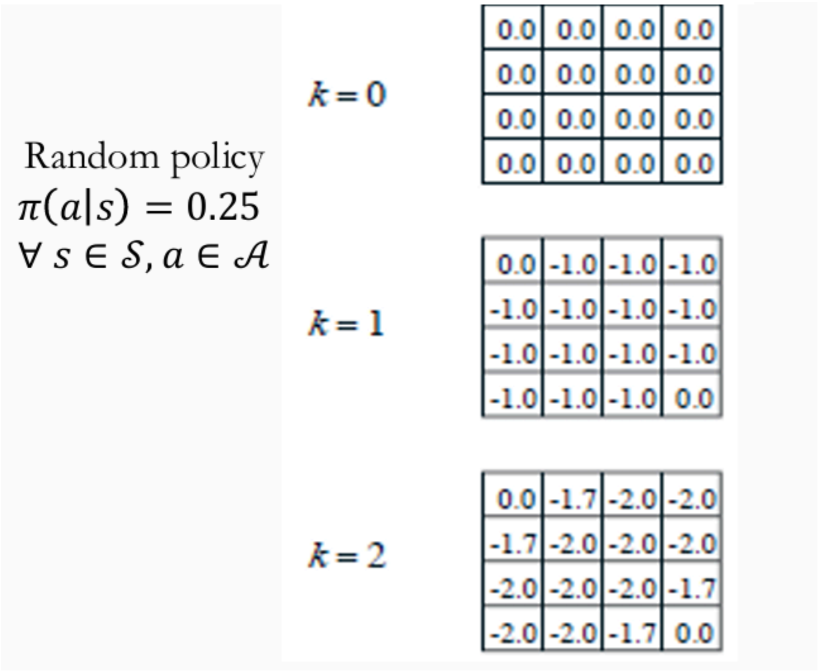
$$\mathbf{v}^{k+1} = \mathcal{R}^\pi + \gamma \mathcal{P}^\pi \mathbf{v}^k$$



Undiscounted episodic MDP ($\gamma = 1$)

$r = -1$ on all transitions

Terminal state is gray



Random policy
$\pi(a|s) = 0.25$
$\forall\, s \in \mathcal{S}, a \in \mathcal{A}$

$k = 0$

0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0

$k = 1$

0.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	0.0

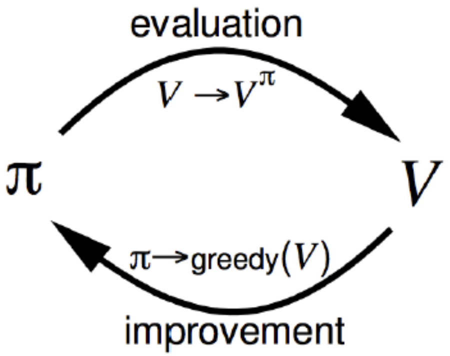
$k = 2$

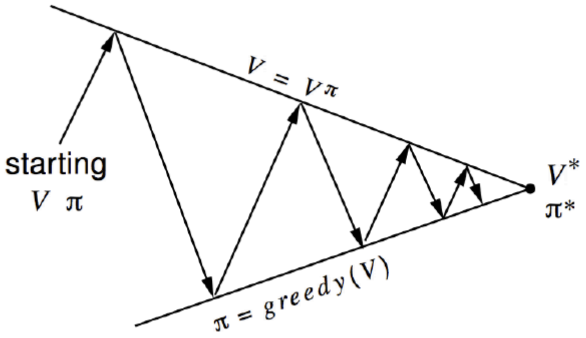
0.0	-1.7	-2.0	-2.0
-1.7	-2.0	-2.0	-2.0
-2.0	-2.0	-2.0	-1.7
-2.0	-2.0	-1.7	0.0

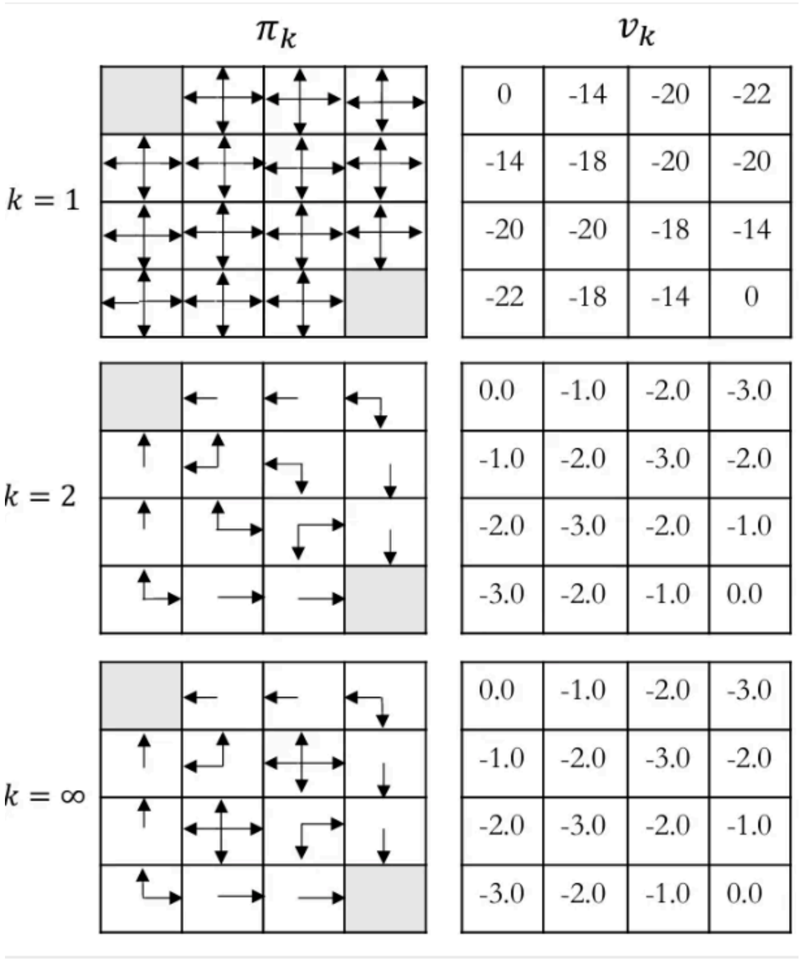
Image Credit: Sutton and Barto book, Example 4.1, pp. 76.

(1.B.1) Policy **Iteration** – How to improve a policy? How to find the optimal policy?

- Given a policy π, find **optimal policy** π **(Control)**
  - Evaluate the policy π, estimate v_π
  - Improve policy by acting **greedily** with respect to v_π
    - $\pi'(s) = \arg\max_a q_\pi(s,a) = \arg\max_a \left(r + \gamma \sum_{s'} p^a_{ss'} v_\pi(s')\right)$
    - $q_\pi(s, \pi'(s)) = \max_a q_\pi(s,a) \geq q_\pi(s, \pi(s)) = v_\pi(s)$
- If improvement stops, we have reached the **optimal policy** (also optimal value function)
  - $q_\pi(s, \pi'(s)) = \max_a q_\pi(s,a) = q_\pi(s, \pi(s)) = v_\pi(s)$
  - Bellman Optimality equation is satisfied
  - $v_\pi(s) = \max_a q_\pi(s,a) = v(s)$ for all s







$k = 1$

0	-14	-20	-22
-14	-18	-20	-20
-20	-20	-18	-14
-22	-18	-14	0

$k = 2$

0.0	-1.0	-2.0	-3.0
-1.0	-2.0	-3.0	-2.0
-2.0	-3.0	-2.0	-1.0
-3.0	-2.0	-1.0	0.0

$k = \infty$

0.0	-1.0	-2.0	-3.0
-1.0	-2.0	-3.0	-2.0
-2.0	-3.0	-2.0	-1.0
-3.0	-2.0	-1.0	0.0

Image Credit: Sutton and Barto.

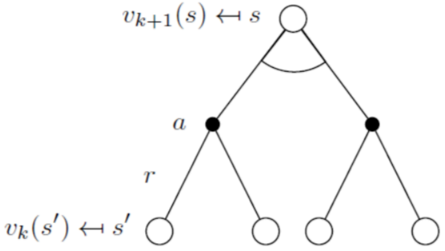
(1.B.2) Value **Iteration** – Estimate optimal value function

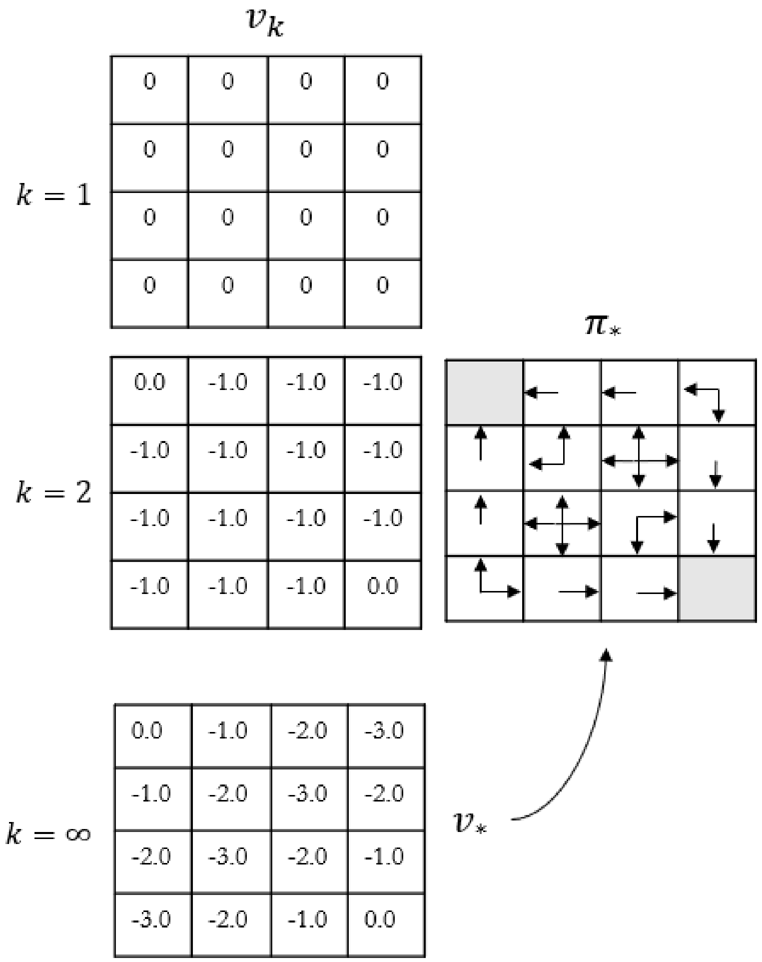
- Find optimal value function v_* **directly** (get optimal policy π_* from v_*).
  - Unlike policy iteration, there is **no explicit policy**.
  - Use **Bellman Optimality equation** to get $v_*(s)$ from the solution to subproblems $v_*(s')$.
- Solution – Iterative application of Bellman optimality equation and dynamic programming.
  - At each iteration $k+1$, update $v_{k+1}(s)$ from $v_k(s')$ for all state s and successor states s'.
  - Bellman optimality update:

$$v_{k+1}(s) = \max_a \sum_{s',r} p(s', r \mid s, a)[r + \gamma v_k(s')]$$

$$v_{k+1}(s) = \max_{a \in \mathcal{A}} \left(\mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_k(s') \right)$$

$$\mathbf{v}_{k+1} = \max_{a \in \mathcal{A}} \boldsymbol{\mathcal{R}}^{\mathbf{a}} + \gamma \boldsymbol{\mathcal{P}}^{\mathbf{a}} \mathbf{v}_k$$





$k = 1$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

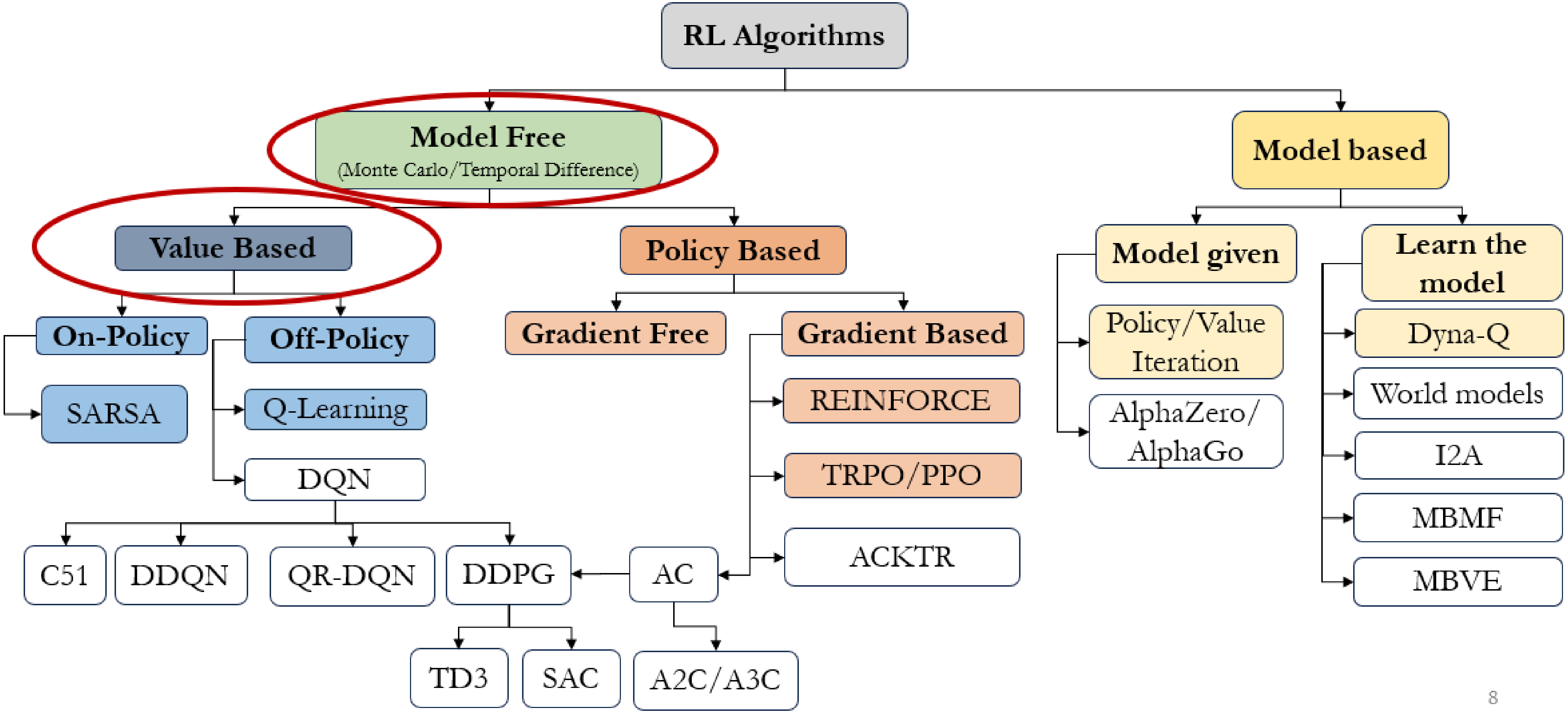
$k = 2$

0.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	0.0

$k = \infty$

0.0	-1.0	-2.0	-3.0
-1.0	-2.0	-3.0	-2.0
-2.0	-3.0	-2.0	-1.0
-3.0	-2.0	-1.0	0.0

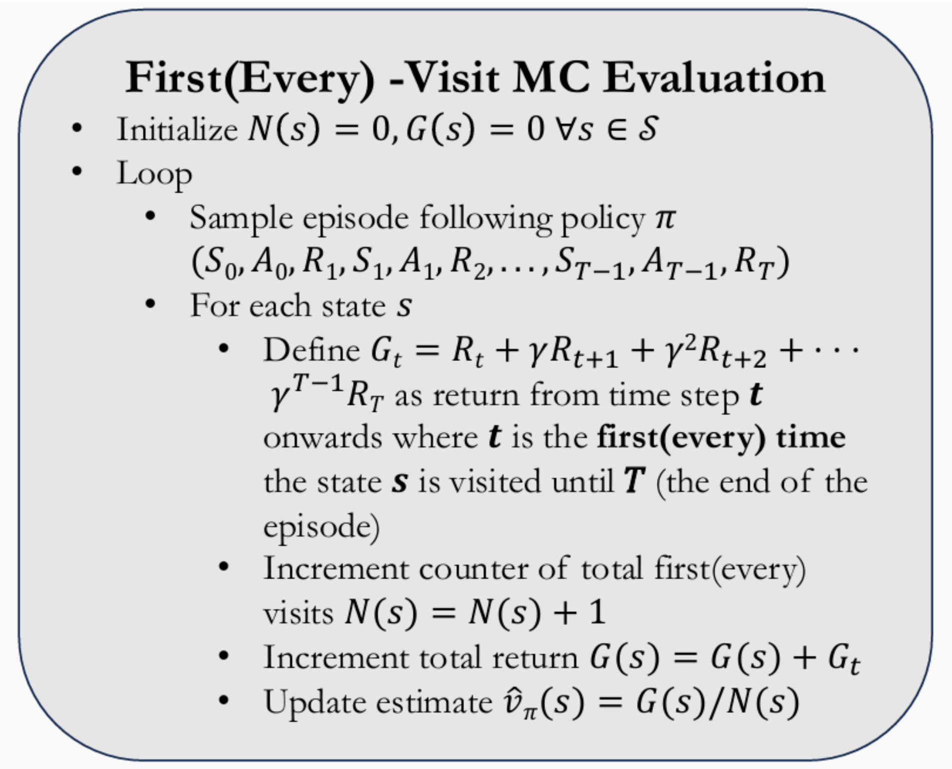
Categorizing RL Algorithms



- **RL Algorithms**
  - **Model Free** (Monte Carlo/Temporal Difference)
    - **Value Based**
      - **On-Policy**
        - SARSA
      - **Off-Policy**
        - Q-Learning
        - DQN
          - C51
          - DDQN
          - QR-DQN
          - DDPG
            - TD3
            - SAC
    - **Policy Based**
      - **Gradient Free**
      - **Gradient Based**
        - REINFORCE
        - TRPO/PPO
        - ACKTR
        - AC
          - DDPG
          - A2C/A3C
  - **Model based**
    - **Model given**
      - Policy/Value Iteration
      - AlphaZero/AlphaGo
    - **Learn the model**
      - Dyna-Q
      - World models
      - I2A
      - MBMF
      - MBVE

(2.A.1) Monte Carlo Policy **Evaluation** – Estimate value function for unknown MDPs (Model Free Prediction)

- No knowledge of MDP transitions or rewards
  - Observe the environment by sampling trajectories.
  - Learn directly from experience (multiple episodes).
- Estimate value function
  - Take the mean of the returns observed.
  - Consider complete episodes.
- Assumptions
  - Applicable to episodic MDPs.
  - All episodes must terminate (**finite horizon MDPs**).

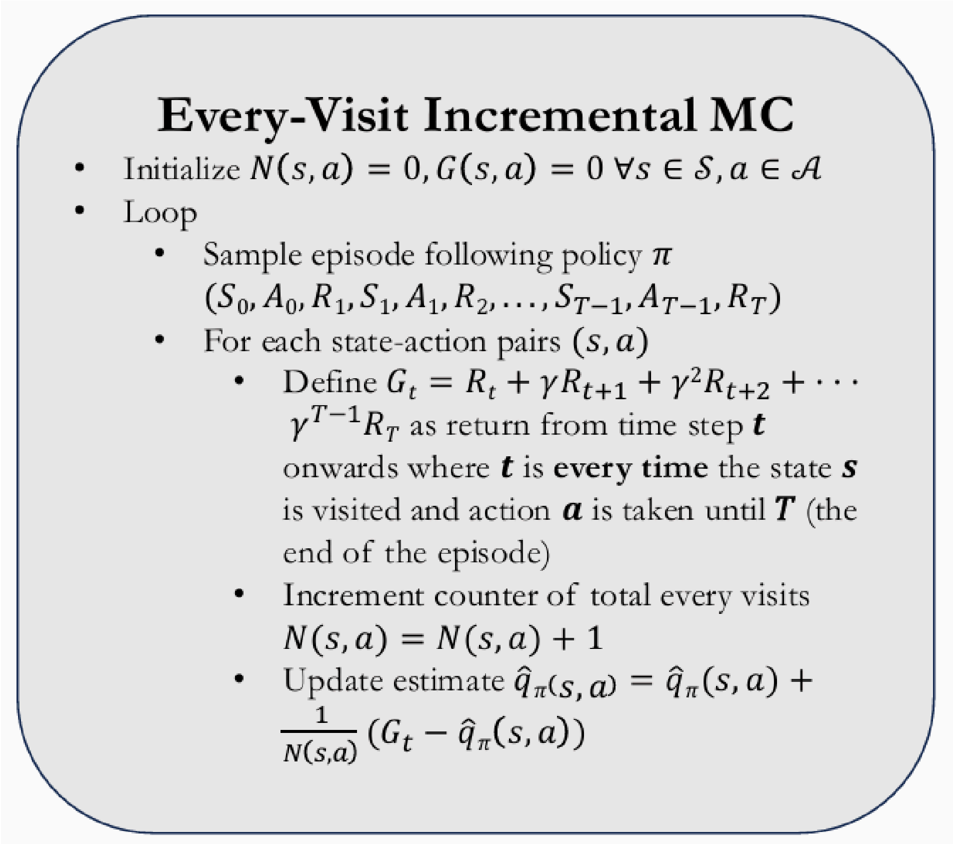


First(Every) -Visit MC Evaluation

- Initialize $N(s) = 0, G(s) = 0 \; \forall s \in \mathcal{S}$
- Loop
  - Sample episode following policy π $(S_0, A_0, R_1, S_1, A_1, R_2, \ldots, S_{T-1}, A_{T-1}, R_T)$
  - For each state s
    - Define $G_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots \gamma^{T-1} R_T$ as return from time step $\boldsymbol{t}$ onwards where $\boldsymbol{t}$ is the **first(every) time** the state $\boldsymbol{s}$ is visited until $\boldsymbol{T}$ (the end of the episode)
    - Increment counter of total first(every) visits $N(s) = N(s) + 1$
    - Increment total return $G(s) = G(s) + G_t$
    - Update estimate $\hat{v}_\pi(s) = G(s)/N(s)$

(2.A.2) Monte Carlo Policy **Evaluation** – Estimate value function for unknown MDPs (Model Free Prediction)

- MC updates can be done **incrementally**
  - Uses formula to calculate incremental mean μ_k of a sequence $x_1, x_2, \dots, x_k$
  - $\mu_k = \mu_{k-1} + \frac{1}{k}\left(x_k - \mu_{k-1}\right)$
  - $\hat{v}_\pi(s) \leftarrow \hat{v}_\pi(s) + \frac{1}{N(s)}\left(G_t - \hat{v}_\pi(s)\right)$
- Estimate **state-action value function** (q)
  - $\hat{q}_\pi(s,a) \leftarrow \hat{q}_\pi(s,a) + \frac{1}{N(s,a)}\left(G_t - \hat{q}_\pi(s,a)\right)$
  - $\hat{q}_\pi(s,a) \leftarrow \hat{q}_\pi(s,a) + \alpha\left(G_t - \hat{q}_\pi(s,a)\right)$, where α can be viewed as **step size** or learning rate
- **Limitations**
  - High variance estimator, require lots of data
  - Episode must end before data from episode can be used to update



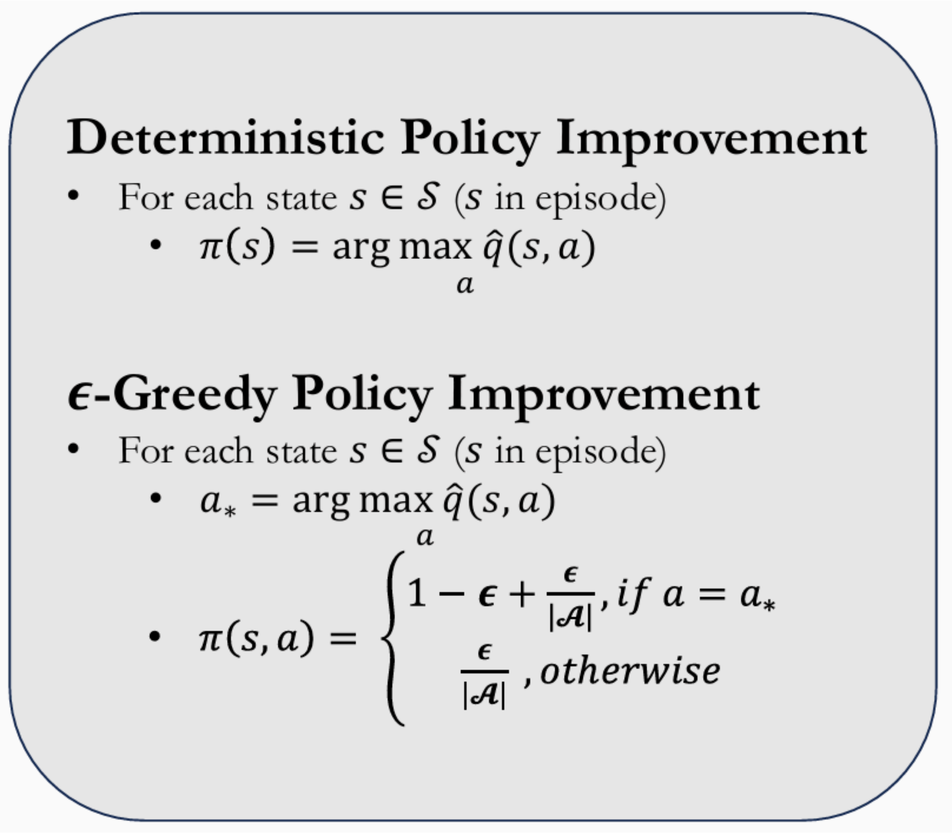
Every-Visit Incremental MC

- Initialize $N(s,a) = 0, G(s,a) = 0 \; \forall s \in \mathcal{S}, a \in \mathcal{A}$
- Loop
  - Sample episode following policy π $(S_0, A_0, R_1, S_1, A_1, R_2, \dots, S_{T-1}, A_{T-1}, R_T)$
  - For each state-action pairs (s,a)
    - Define $G_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots \gamma^{T-1} R_T$ as return from time step $\boldsymbol{t}$ onwards where $\boldsymbol{t}$ is **every time** the state $\boldsymbol{s}$ is visited and action $\boldsymbol{a}$ is taken until $\boldsymbol{T}$ (the end of the episode)
    - Increment counter of total every visits $N(s,a) = N(s,a) + 1$
    - Update estimate $\hat{q}_\pi(s,a) = \hat{q}_\pi(s,a) + \frac{1}{N(s,a)}\left(G_t - \hat{q}_\pi(s,a)\right)$

(2.B) Monte Carlo Policy **Optimization** – Estimate optimal value function

for unknown MDPs (Model Free Control)

- No knowledge of **MDP transitions or rewards**
    - Observe the environment by sampling trajectories.
    - Learn directly from experience (multiple episodes).
- Estimate the **optimal value function**
    - Use **Policy Iteration** approach.
    - MC method in policy evaluation step.
    - Greedy policy improvement on action-value function q.
    - $\pi'(s) = \arg\max_a q(s,a)$
- **Caveats**
    - Greedy policy improvement on state value function (v) not possible, requires MDP model (i.e., only applicable to action-value function q).
    - Might not explore all states – can be solved using **stochastic policy (ϵ-greedy)** to encourage continuous exploration.



Deterministic Policy Improvement

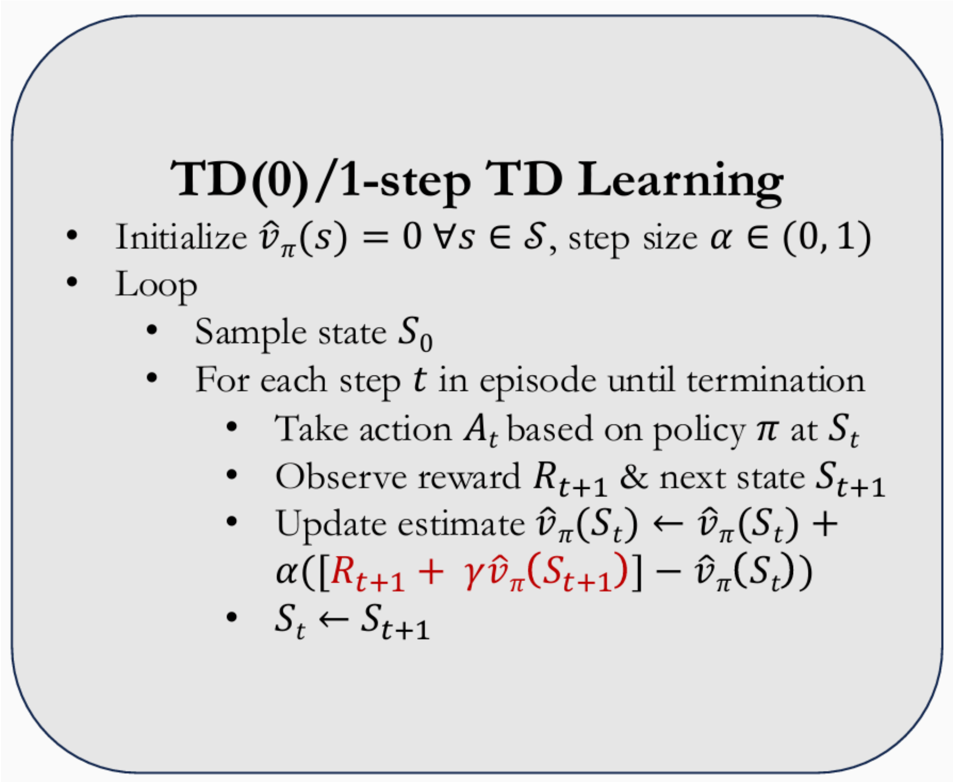
- For each state $s \in \mathcal{S}$ (s in episode)
    - $\pi(s) = \arg\max_a \hat{q}(s,a)$

ϵ-Greedy Policy Improvement

- For each state $s \in \mathcal{S}$ (s in episode)
    - $a_* = \arg\max_a \hat{q}(s,a)$
    - $\pi(s,a) = \begin{cases} 1 - \epsilon + \frac{\epsilon}{|\mathcal{A}|}, if\ a = a_* \\ \frac{\epsilon}{|\mathcal{A}|}, otherwise \end{cases}$

(3.A) Temporal Difference(TD) **Learning** – Estimate value function for unknown MDPs (Model Free Prediction)

- Combination of **Monte Carlo** & **dynamic programming** methods
  - Immediately update estimate of v after each observed (s, a, r, s') tuple
  - TD learns from **incomplete episodes**, by bootstrapping
- **Estimate value function**
  - Update value toward estimated target return
  - TD target: $R_{t+1} + \gamma \hat{v}(S_{t+1})$
  - TD error: $\delta_t = [R_{t+1} + \gamma \hat{v}(S_{t+1})] - \hat{v}(S_t)$
- **Advantages**
  - Lower variance than MC (although biased estimator)
  - Can be used in episodic or infinite-horizon non-episodic **MDPs**

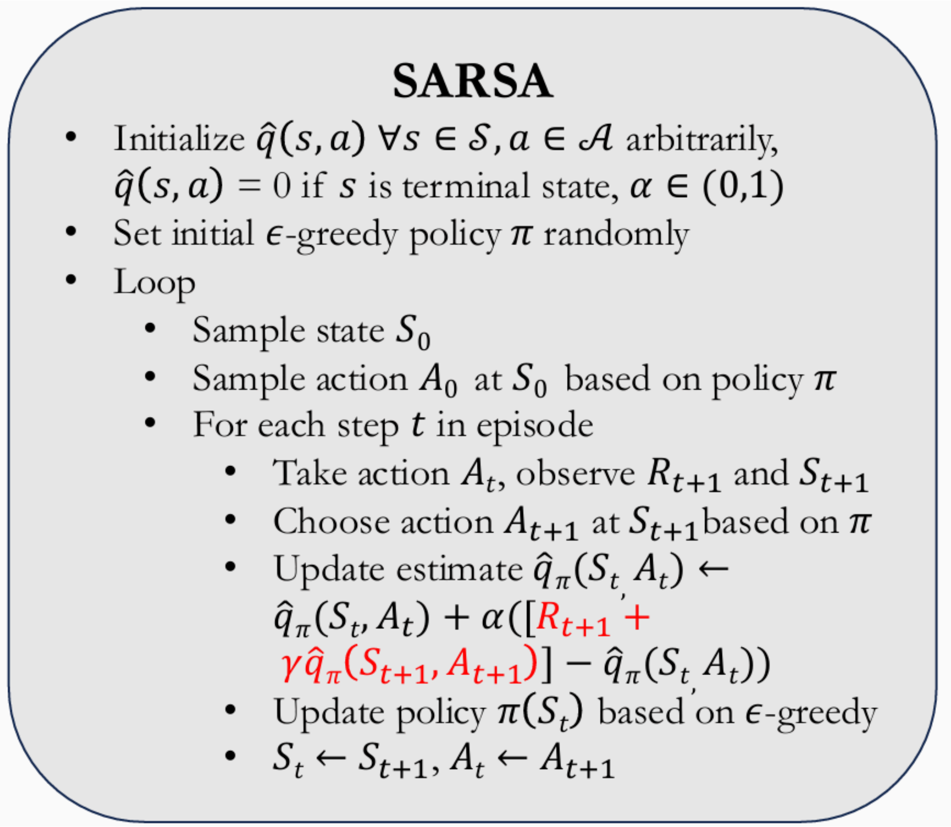


TD(0)/1-step TD Learning

- Initialize $\hat{v}_\pi(s) = 0 \ \forall s \in \mathcal{S}$, step size $\alpha \in (0, 1)$
- Loop
  - Sample state S_0
  - For each step t in episode until termination
    - Take action A_t based on policy π at S_t
    - Observe reward R_{t+1} & next state S_{t+1}
    - Update estimate $\hat{v}_\pi(S_t) \leftarrow \hat{v}_\pi(S_t) + \alpha([R_{t+1} + \gamma \hat{v}_\pi(S_{t+1})] - \hat{v}_\pi(S_t))$
    - $S_t \leftarrow S_{t+1}$

(3.B.1) Model-Free Control with TD Methods **SARSA** – (On-Policy TD Learning)

- Uses **TD learning** approach for policy evaluation
  - Estimate q of the policy π being followed
  - ϵ-Greedy policy improvement on action-value function q
- **Estimate action value function**
  - Update value toward estimated target return given $(S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1})$ transition tuple (hence called **SARSA**)
  - SARSA target: $R_{t+1} + \gamma\hat{q}_\pi(S_{t+1}, A_{t+1})$
- **Advantages**
  - **On-policy** algorithm
  - Converges to the optimal action-value function, $\hat{q}_\pi(s, a) \to q_*(s, a)$



SARSA

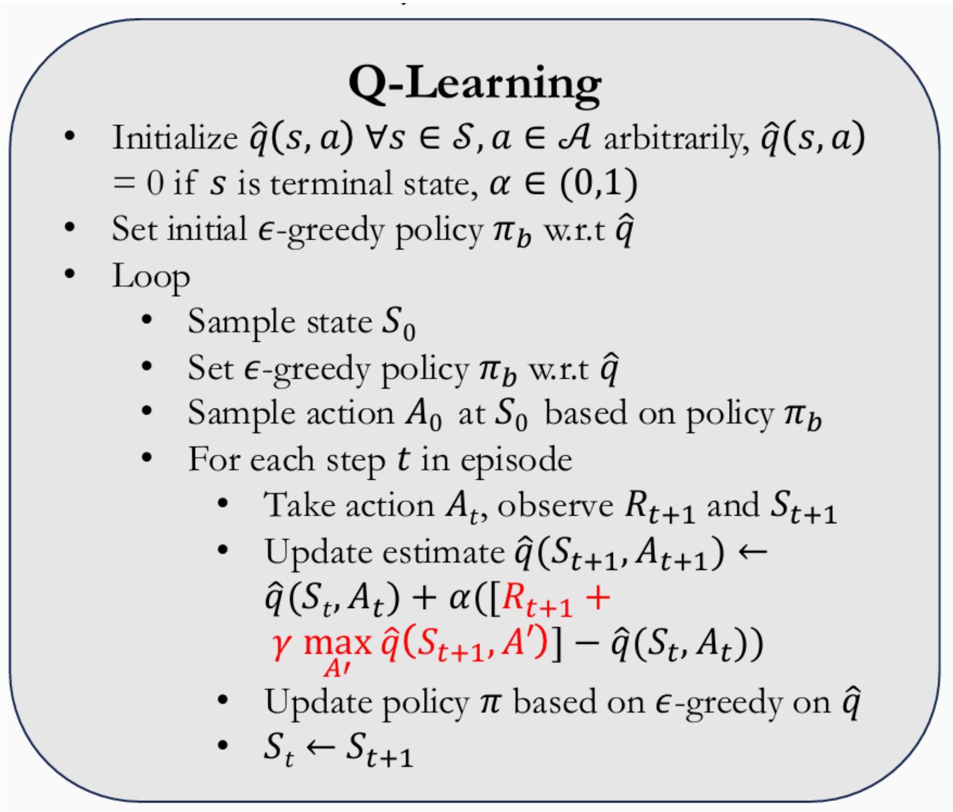
- Initialize $\hat{q}(s, a)\ \forall s \in \mathcal{S}, a \in \mathcal{A}$ arbitrarily, $\hat{q}(s, a) = 0$ if s is terminal state, $\alpha \in (0,1)$
- Set initial ϵ-greedy policy π randomly
- Loop
  - Sample state S_0
  - Sample action A_0 at S_0 based on policy π
  - For each step t in episode
    - Take action A_t, observe R_{t+1} and S_{t+1}
    - Choose action A_{t+1} at S_{t+1} based on π
    - Update estimate $\hat{q}_\pi(S_t, A_t) \leftarrow \hat{q}_\pi(S_t, A_t) + \alpha([R_{t+1} + \gamma\hat{q}_\pi(S_{t+1}, A_{t+1})] - \hat{q}_\pi(S_t, A_t))$
    - Update policy $\pi(S_t)$ based on ϵ-greedy
    - $S_t \leftarrow S_{t+1}, A_t \leftarrow A_{t+1}$

On-policy versus Off-Policy Learning & Control

- **On-policy learning**
  - Learn to estimate and evaluate a policy π from experience obtained from following that policy (**same policy for prediction and control**).
  - Direct experience
- **Off-policy learning**
  - Learn to estimate and evaluate a policy π^t (called **target policy**) using experience gathered from following a different policy (called **behavior policy** π^b).
  - Indirect experience, learn from observing humans or other agents
  - Re-use experience generated from old policies
  - Learn about optimal policy while following exploratory policy
  - Learn about multiple policies while following one policy
- Need **importance sampling** corrections on returns along whole episode
  - $G_t^{\pi^t/\pi^b} = \left(\frac{\pi^t(A_t|S_t)}{\pi^b(A_t|S_t)} \frac{\pi^t(A_{t+1}|S_{t+1})}{\pi^b(A_{t+1}|S_{t+1})} \cdots \frac{\pi^t(A_T|S_T)}{\pi^b(A_T|S_T)} \right) G_t$

(3.B.2) Model-Free Control with TD Methods – Q Learning (Off-Policy TD Learning)

- Q-learning is an **off-policy** RL algorithm on action-values q
- **Maintain state-action q estimates for bootstrapping**
  - Use the value of the best future action
  - Stochastic approximation like SARSA
- **Estimate action value function**
  - Next action is chosen using behavior policy $A_{t+1} \sim \pi_b(S_t)$
  - Consider all alternative successor action $A' \sim \pi(S_t)$, take best A' for update
  - Q-learning target:
    $R_{t+1} + \gamma \max_{A'} \hat{q}(S_{t+1}, A')$
- **Advantages**
  - No importance sampling required
  - Allows both behavior and target policies to improve



Q-Learning

```
• Initialize $\hat{q}(s,a)\ \forall s \in \mathcal{S}, a \in \mathcal{A}$ arbitrarily, $\hat{q}(s,a) = 0$ if $s$ is terminal state, $\alpha \in (0,1)$
• Set initial $\epsilon$-greedy policy $\pi_b$ w.r.t $\hat{q}$
• Loop
    • Sample state $S_0$
    • Set $\epsilon$-greedy policy $\pi_b$ w.r.t $\hat{q}$
    • Sample action $A_0$ at $S_0$ based on policy $\pi_b$
    • For each step $t$ in episode
        • Take action $A_t$, observe $R_{t+1}$ and $S_{t+1}$
        • Update estimate $\hat{q}(S_{t+1}, A_{t+1}) \leftarrow \hat{q}(S_t, A_t) + \alpha([R_{t+1} + \gamma \max_{A'} \hat{q}(S_{t+1}, A')] - \hat{q}(S_t, A_t))$
        • Update policy $\pi$ based on $\epsilon$-greedy on $\hat{q}$
        • $S_t \leftarrow S_{t+1}$
```

(4.A) Value Function Approximation – Scaling up RL methods

- So far, we have been working with the tabular representation of the value functions $v(s)$ or $q(s, a)$ and policy $\pi(a \mid s)$ for finite and discrete MDPs
- But MDPs can be very large, need to **scale up** for large MDPs
  - Too many states and/or actions to store in memory, state space can be continuous
  - Too slow to learn the value of each state individually
- Solution – Estimate value function with **function approximation**
  - $\hat{v}(s, \theta) \approx v_\pi(s)$ or $\hat{q}(s, a, \theta) \approx q_\pi(s, a)$ where the value function is parameterized by θ
  - Update parameter θ using MC and TD methods (supervised learning)
  - Generalizes to unseen states and/or actions
- Common Function Approximators (consider only differentiable ones)
  - Linear combination of features • Nearest Neighbors
  - Neural Networks • Decision Trees

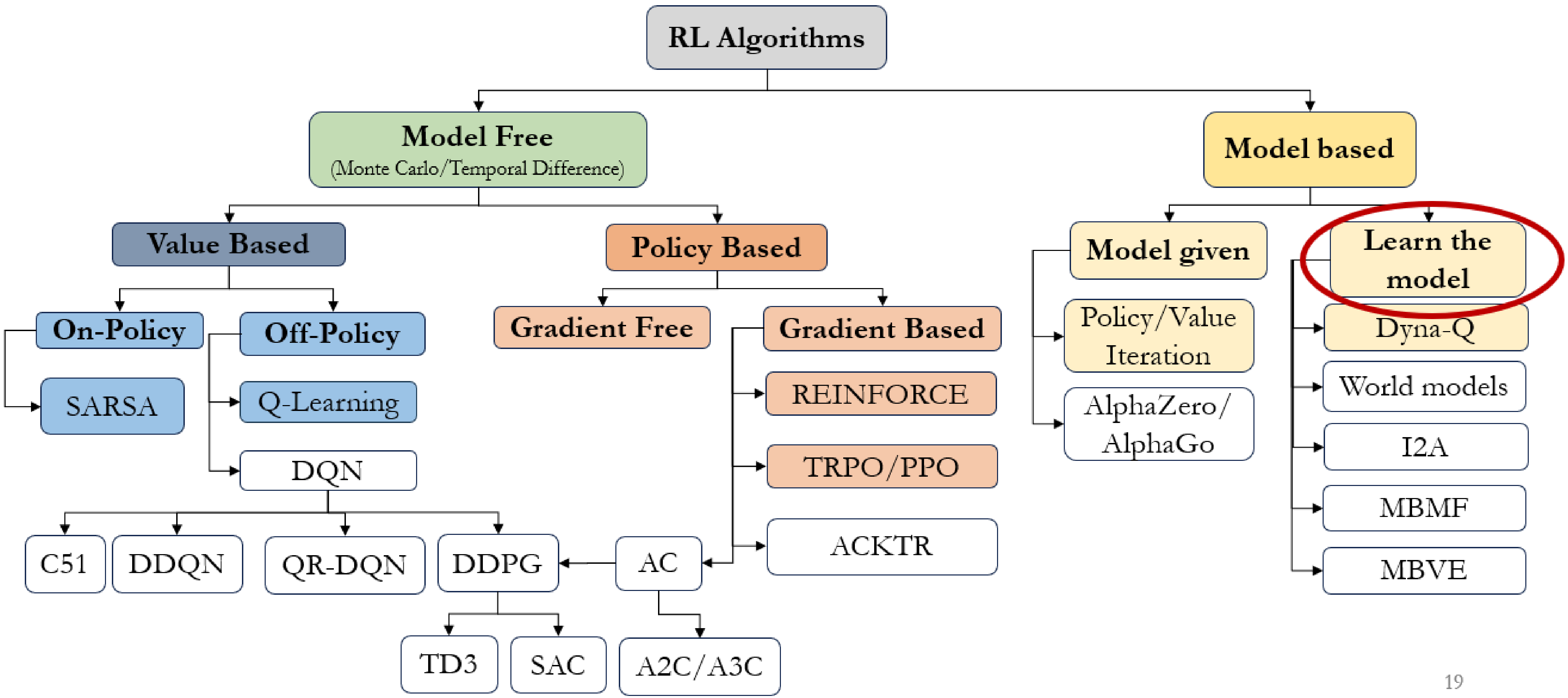
(4.A.1) Linear Value Function Approx. by Gradient Descent

- Represent state by a feature vector $\mathbf{x}(s) = [x_1(s), x_2(s), \ldots, x_n(s)]^T$
- Represent value function by a **linear combination of features**
  - $\hat{v}(s, \boldsymbol{\theta}) = \mathbf{x}(s)^T \boldsymbol{\theta}$, where $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_n]^T$
- Find parameter vector $\boldsymbol{\theta}$ minimizing the mean-squared error between approximate value function $\hat{v}(s, \boldsymbol{\theta})$ and true value function $v_\pi(s)$ (value objective function)
  - $J(\boldsymbol{\theta}) = \mathbb{E}_\pi\big[(v_\pi(s) - \hat{v}(s, \boldsymbol{\theta}))^2\big]$
  - $J_{\text{linear}}(\boldsymbol{\theta}) = \mathbb{E}_\pi\big[(v_\pi(s) - \mathbf{x}(s)^T \boldsymbol{\theta})^2\big]$ (for linear value function approx.)
- Apply **gradient descent** (or SGD) to find local minimum by updating parameters
  - Update rule: $\Delta\boldsymbol{\theta} = -\frac{1}{2}\alpha \nabla J(\boldsymbol{\theta}) = \alpha\, \mathbb{E}_\pi\big[(v_\pi(s) - \hat{v}(s, \boldsymbol{\theta}))\, \nabla_{\boldsymbol{\theta}} \hat{v}(s, \boldsymbol{\theta})\big]$
  - SGD update rule: $\Delta\boldsymbol{\theta} = \alpha\big[(v_\pi(s) - \hat{v}(s, \boldsymbol{\theta}))\, \nabla_{\boldsymbol{\theta}} \hat{v}(s, \boldsymbol{\theta})\big]$
  - SGD update rule for **linear** value function approx.: $\Delta\boldsymbol{\theta} = \alpha\big[(v_\pi(s) - \hat{v}(s, \boldsymbol{\theta}))\, \mathbf{x}(s)\big]$
- Stochastic gradient descent converges to global optimum
- Seems great… but we don’t know v_π!

(4.A.1) Incremental Prediction/Control Algorithm – MC/TD with Function Approx.

- In practice, we don't have true value function v_π for prediction, we only have rewards through environment interaction, thus **substitute target for** v_π
    - For MC, the target is the return G_t
        - $\Delta\theta = \alpha\left[\left(G_t - \hat{v}(S_t,\theta)\right)\nabla_\theta\hat{v}(S_t,\theta)\right]$
    - For TD(0), the target is the TD target $R_{t+1} + \gamma\hat{v}(S_{t+1},\theta)$
        - $\Delta\theta = \alpha\left[\left(R_{t+1} + \gamma\hat{v}(S_{t+1},\theta) - \hat{v}(S_t,\theta)\right)\nabla_\theta\hat{v}(S_t,\theta)\right]$
- In control, approximate action-value function $\hat{q}(s,a,\theta)$, substitute target for true value of q_π
    - For MC, the target is the return G_t
        - $\Delta\theta = \alpha\left[\left(G_t - \hat{q}(S_t,A_t,\theta)\right)\nabla_\theta\hat{q}(S_t,A_t,\theta)\right]$
    - For TD(0), the target is the TD target $R_{t+1} + \gamma\hat{q}(S_{t+1},A_{t+1},\theta)$
        - $\Delta\theta = \alpha\left[\left(R_{t+1} + \gamma\hat{q}(S_{t+1},A_{t+1},\theta) - \hat{q}(S_t,A_t,\theta)\right)\nabla_\theta\hat{q}(S_t,A_t,\theta)\right]$
- (4.B) Approximate Policy Iteration – Do approximate policy evaluation using $\hat{q}(s,a,\theta) \approx q_\pi$ followed by ϵ-greedy policy improvement

Categorizing RL Algorithms



- **RL Algorithms**
  - **Model Free** (Monte Carlo/Temporal Difference)
    - **Value Based**
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        - SARSA
      - **Off-Policy**
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        - DQN
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          - DDQN
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          - DDPG
            - TD3
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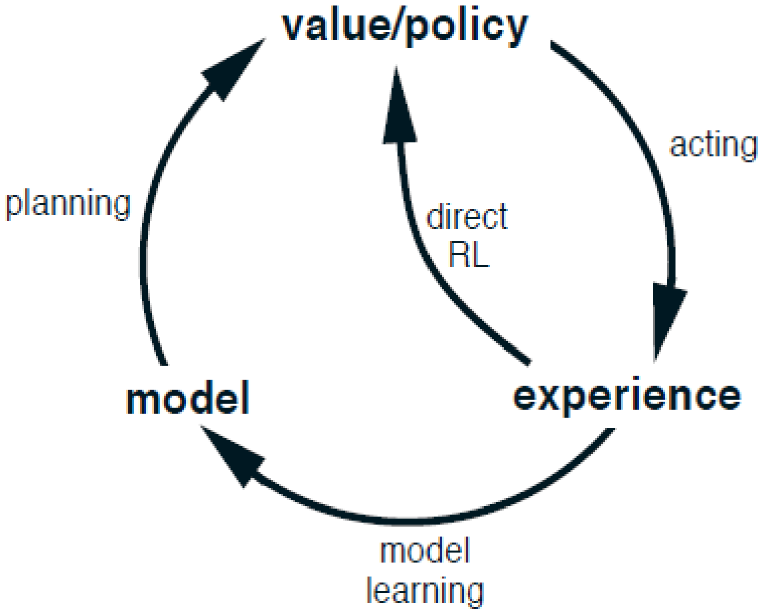
Model-Based Reinforcement Learning – Integrating Learning and Planning

- Previous approach – **Model Free RL**
  - **No model** (unknown transition function $\mathcal{P}$ and reward function $\mathcal{R}$)
  - Learn value function/policy directly from experience
- New Approach – **Model Based RL**
  - First **learn (estimate) model** from experience
  - Plan for optimal value function/policy using learned model
  - Integrate learning and planning into a single architecture
  - Possible to efficiently learn model using supervised learning methods
  - Can understand model uncertainty
  - Model-based RL is only as good as the estimated model.
    When the model is inaccurate, planning process will compute a suboptimal policy.



$$\text{Model } \mathcal{M}_\eta \xrightarrow{\text{represents}} \text{MDP } \langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$$

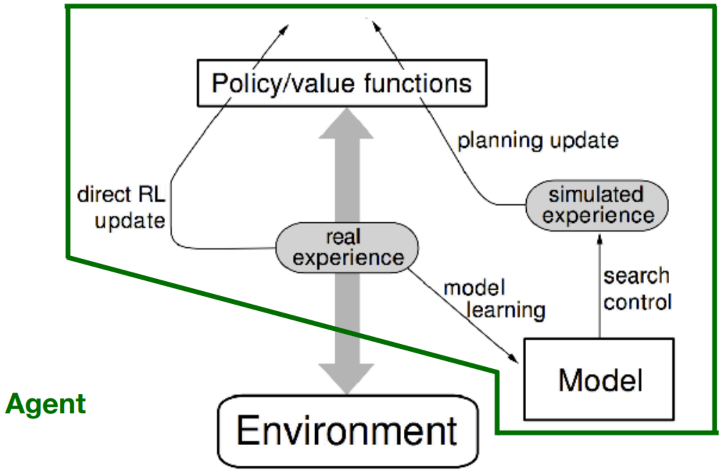
$$\mathcal{M}_\eta = \langle \mathcal{P}_\eta, \mathcal{R}_\eta \rangle \quad (\eta \text{ is the parameter})$$

$$\mathcal{P}_\eta \approx \mathcal{P} \qquad \mathcal{R}_\eta \approx \mathcal{R}$$

(5.A/B) Integrated Architectures – Dyna (Dyna-Q Algorithm)

- **Dyna**
  - Learn model from real experience
  - Learn and plan value function/policy from both **real & simulated experience** (Q-Learning)
- Involves one-step interaction (acting) with the environment and n steps planning
- Store experience, get better policy with fewer environment interactions

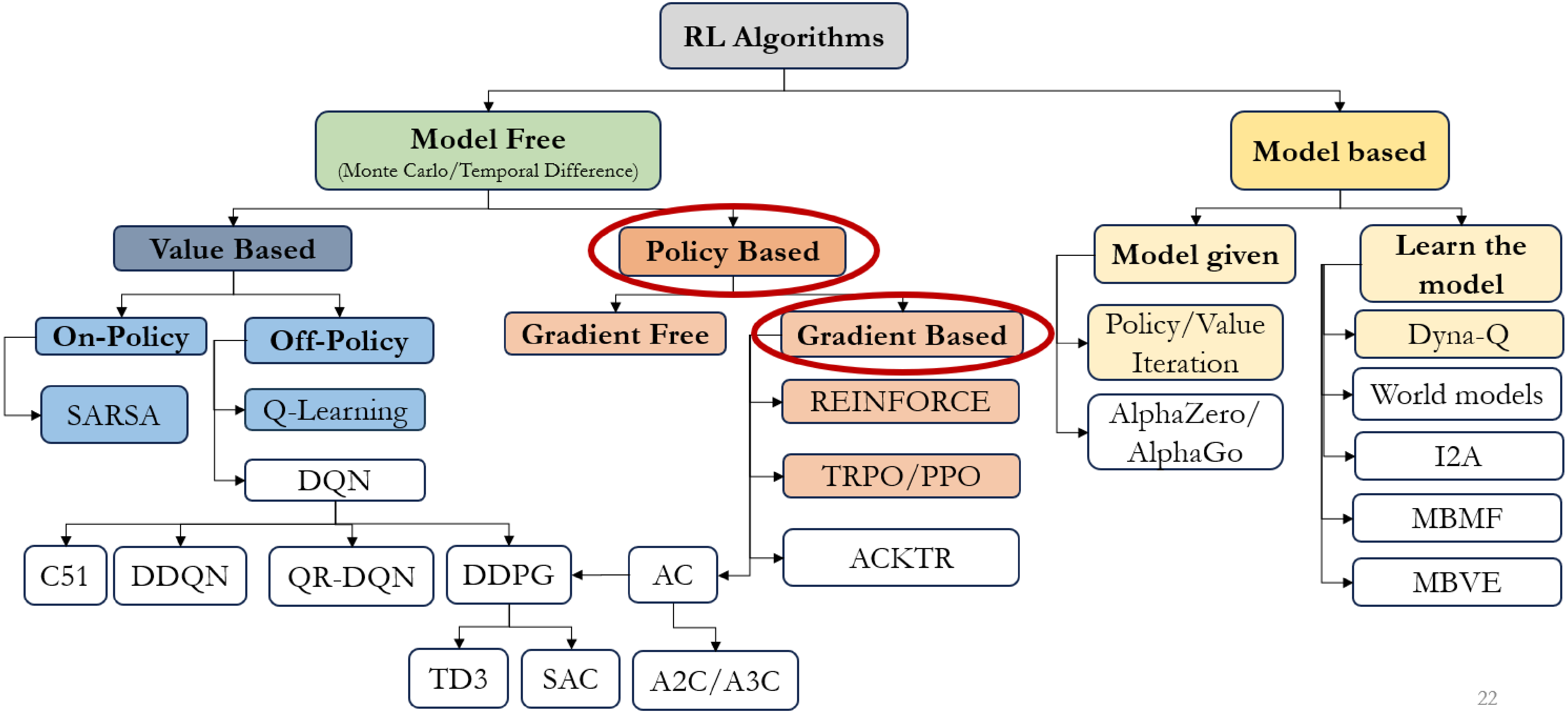
The Dyna Architecture



Tabular Dyna-Q

```
- Initialize $\hat{q}(s,a)$ and $\mathcal{M}(s,a)$ $\forall s \in \mathcal{S}, a \in \mathcal{A}$
- Loop
    - Sample current state $S_t$
    - Sample action $A_t$ at $S_t$ based on $\epsilon$-greedy on $\hat{q}$
    - Take action $A_t$, observe $R_{t+1}$ and $S_{t+1}$
    - $\hat{q}(S_{t+1}, A_t) \leftarrow \hat{q}(S_t, A_t) + \alpha([R_{t+1} + \gamma \max_{A'} \hat{q}(S_{t+1}, A')] - \hat{q}(S_t, A_t))$
    - $\mathcal{M}(S_t, A_t) \leftarrow R_{t+1}, S_{t+1}$
    - Loop $n$ times
        - Sample random state s
        - Sample random previous action $a$ at $s$
        - $r, s' \leftarrow \mathcal{M}(s,a)$
        - $\hat{q}(s,a) \leftarrow \hat{q}(s,a) + \alpha([r + \gamma \max_{a'} \hat{q}(s',a')] - \hat{q}(s,a))$
```

Categorizing RL Algorithms



RL Algorithms

Model Free
(Monte Carlo/Temporal Difference)

Model based

Value Based

Policy Based

Model given

Learn the model

On-Policy

Off-Policy

Gradient Free

Gradient Based

SARSA

Q-Learning

DQN

REINFORCE

TRPO/PPO

ACKTR

Policy/Value Iteration

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MBVE

C51

DDQN

QR-DQN

DDPG

AC

TD3

SAC

A2C/A3C

Policy-Based RL – Policy Gradient Methods

- Previously, we approximated the value functions using parameters $\boldsymbol{\theta}$
    - Obtained policy from value function $\hat{v}(s, \boldsymbol{\theta})$ or $\hat{q}(s, a, \boldsymbol{\theta})$ using ϵ-greedy
- Now, directly parameterize and learn the policy $\pi_{\boldsymbol{\theta}}(s, a) = \mathbb{P}[a \mid s, \boldsymbol{\theta}]$
    - Model-Free RL, better convergence properties, can learn stochastic policies
    - Effective in high-dimensional or continuous action spaces
    - Typically converge to a local rather than global optimum
    - Evaluating a policy is typically inefficient and high variance
- Given a policy $\pi_{\boldsymbol{\theta}}(s, a)$ with parameters $\boldsymbol{\theta}$, find best $\boldsymbol{\theta}$ which maximizes $J(\boldsymbol{\theta})$
    - **Policy Objective Function** $J(\boldsymbol{\theta})$ – measures quality of policy $\pi_{\boldsymbol{\theta}}$
        - Episodic environments: $J(\boldsymbol{\theta}) = v_{\pi_{\boldsymbol{\theta}}}(s_1, \boldsymbol{\theta})$ (also called start value)
        - Continuing environments: $J(\boldsymbol{\theta}) = \sum_s d_{\pi_{\boldsymbol{\theta}}}(s)\, v_{\pi_{\boldsymbol{\theta}}}(s, \boldsymbol{\theta})$ (also called average value), where $d_{\pi_{\boldsymbol{\theta}}}(s)$ is the stationary distribution of the Markov chain for $\pi_{\boldsymbol{\theta}}$
- Can use gradient free optimization, but greater efficiency possible using gradient
- **Policy Gradient Methods:**
    - Search for local maximum by **ascending** the policy gradient with $\boldsymbol{\theta}$:

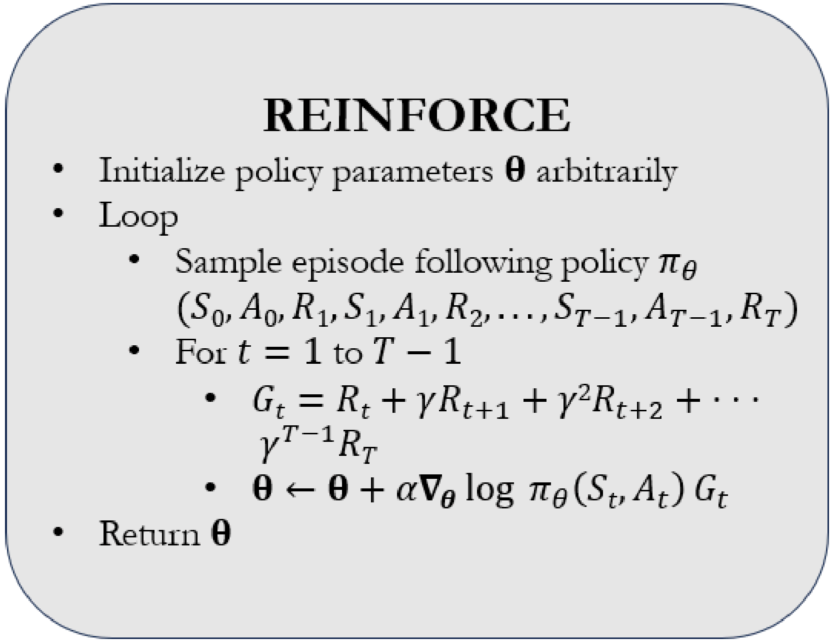
$$\Delta\boldsymbol{\theta} = \alpha \nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta})$$

(6.B) Monte Carlo Policy Gradient – REINFORCE

- **Policy Gradient Theorem**
  - For any differentiable policy

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(S_t, A_t)\, q_{\pi_\theta}(S_t, A_t)\right]$$

  - $\nabla_\theta \log \pi_\theta(s, a)$ is called the **score function**
  - Key observations
    - It allows gradients of policy instead of value.
    - The action value q_{π_θ} can be approximated.
  - Many choices of differentiable policy π_θ – Softmax, Gaussian, Neural Networks
- **Monte Carlo Policy Gradient**
  - Update parameters by stochastic gradient ascent, use policy gradient theorem
  - Use return G_t as an unbiased estimate of $q_{\pi_\theta}(S_t, A_t)$
  - $\Delta\theta = \alpha \nabla_\theta \log \pi_\theta(S_t, A_t)\, G_t$
- **MC policy gradient has high variance**
  - Use actor-critic methods to reduce variance



REINFORCE

```
• Initialize policy parameters $\boldsymbol{\theta}$ arbitrarily
• Loop
    • Sample episode following policy $\pi_\theta$
      $(S_0, A_0, R_1, S_1, A_1, R_2, \ldots, S_{T-1}, A_{T-1}, R_T)$
    • For $t = 1$ to $T - 1$
        • $G_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \cdots \gamma^{T-1} R_T$
        • $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \boldsymbol{\nabla}_{\boldsymbol{\theta}} \log \pi_\theta(S_t, A_t)\, G_t$
• Return $\boldsymbol{\theta}$
```

(7.B) Advanced Policy Gradient Algorithms – Trust Region Methods (TRPO/PPO)

- General policy gradient algorithms try to solve the optimization problem

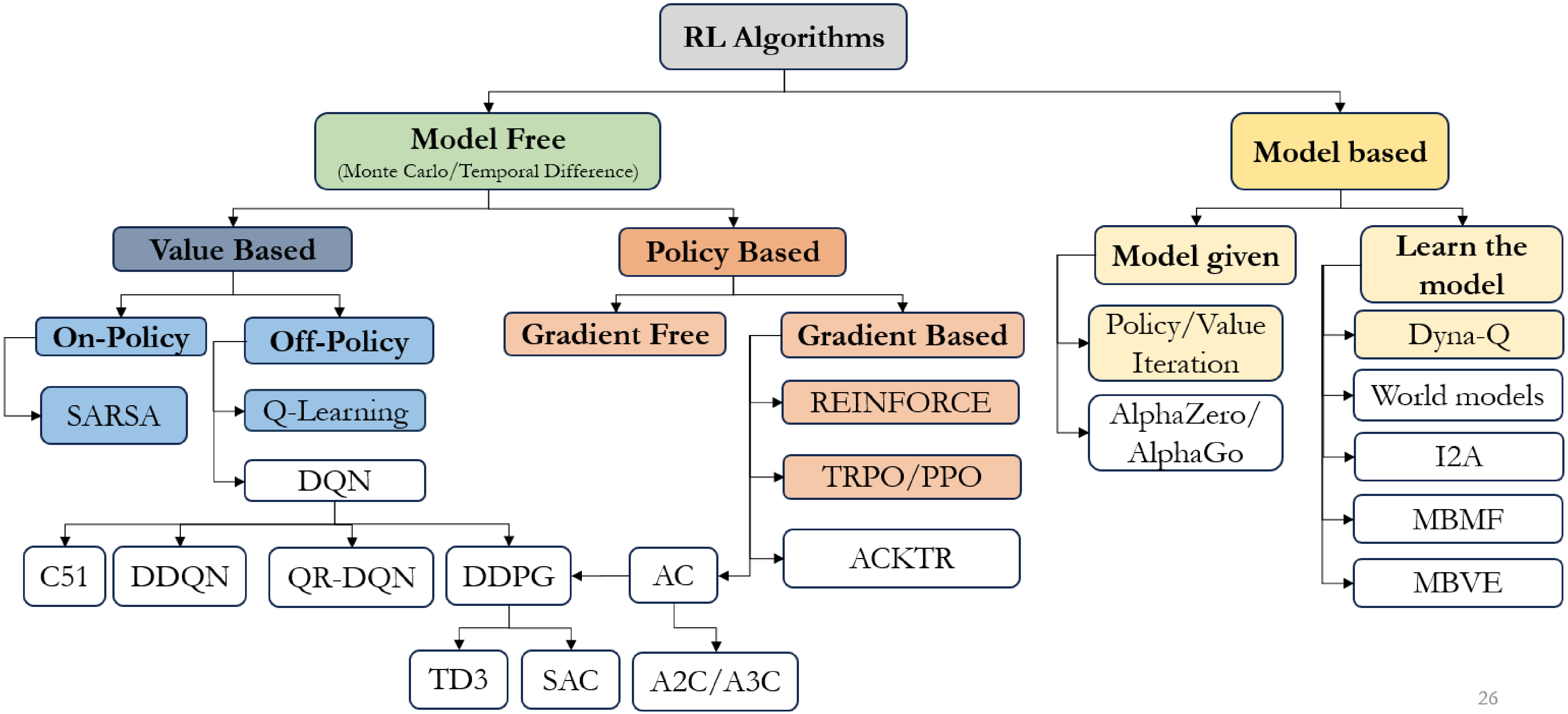
$$\max_{\theta} J(\pi_\theta) = \mathbb{E}_{\tau\sim\pi_\theta}\left[\sum_{t=0}^{\infty}\gamma^t R_t\right]$$

- Use stochastic gradient ascent on policy parameters θ using policy gradient g
  - $g = \nabla_\theta J(\pi_\theta) = \mathbb{E}_{\tau\sim\pi_\theta}[\sum_{t=0}^{\infty}\gamma^t \nabla_\theta \log \pi_\theta(A_t \mid S_t)\, A_{\pi_\theta}(S_t, A_t)]$
  - Advantage function $A_{\pi_\theta}(s,a) = q_{\pi_\theta}(s,a) - v_{\pi_\theta}(s)$, relative **advantage** of an action (how much better to take action a in state s over randomly selecting any other action and following π_θ)
- However, its sample efficiency is poor as it searches in *parameter space* instead of policy space.
  Also, the method is dependent on step size.
- **Trust Region Methods – Proximal Policy Optimization (PPO)**
  - Define $L_\pi(\pi') \approx J(\pi') - J(\pi)$ (π' → new policy, π → old policy), improvement over old policy.
  - Update θ incrementally, approximately penalizing policies for changing too much between steps.
  - **Adaptive KL Penalty:**
    $\theta_{k+1} = \arg\max_\theta \left(L_\theta(\theta) - \beta_k \mathrm{KL}(\theta \| \theta_k)\right)$,
    where β_k is the penalty coefficient.
  - **Clipped Objective:**
    $\theta_{k+1} = \arg\max_\theta L^{\mathrm{CLIP}}_{\theta_k}(\theta)$ where

$$L^{\mathrm{CLIP}}_{\theta_k}(\theta) = \mathbb{E}_{\tau\sim\pi_k}\left[\sum_{t=0}^{T}\min\left(r_t(\theta)\,\hat{A}_{\pi_k}(S_t, A_t), \mathrm{clip}(r_t(\theta), 1-\varepsilon, 1+\varepsilon)\,\hat{A}_{\pi_k}(S_t, A_t)\right)\right],$$

$r_t(\theta) = \dfrac{\pi_\theta(A_t \mid S_t)}{\pi_{\theta_k}(A_t \mid S_t)}$, and ε is a hyperparameter.

Categorizing RL Algorithms



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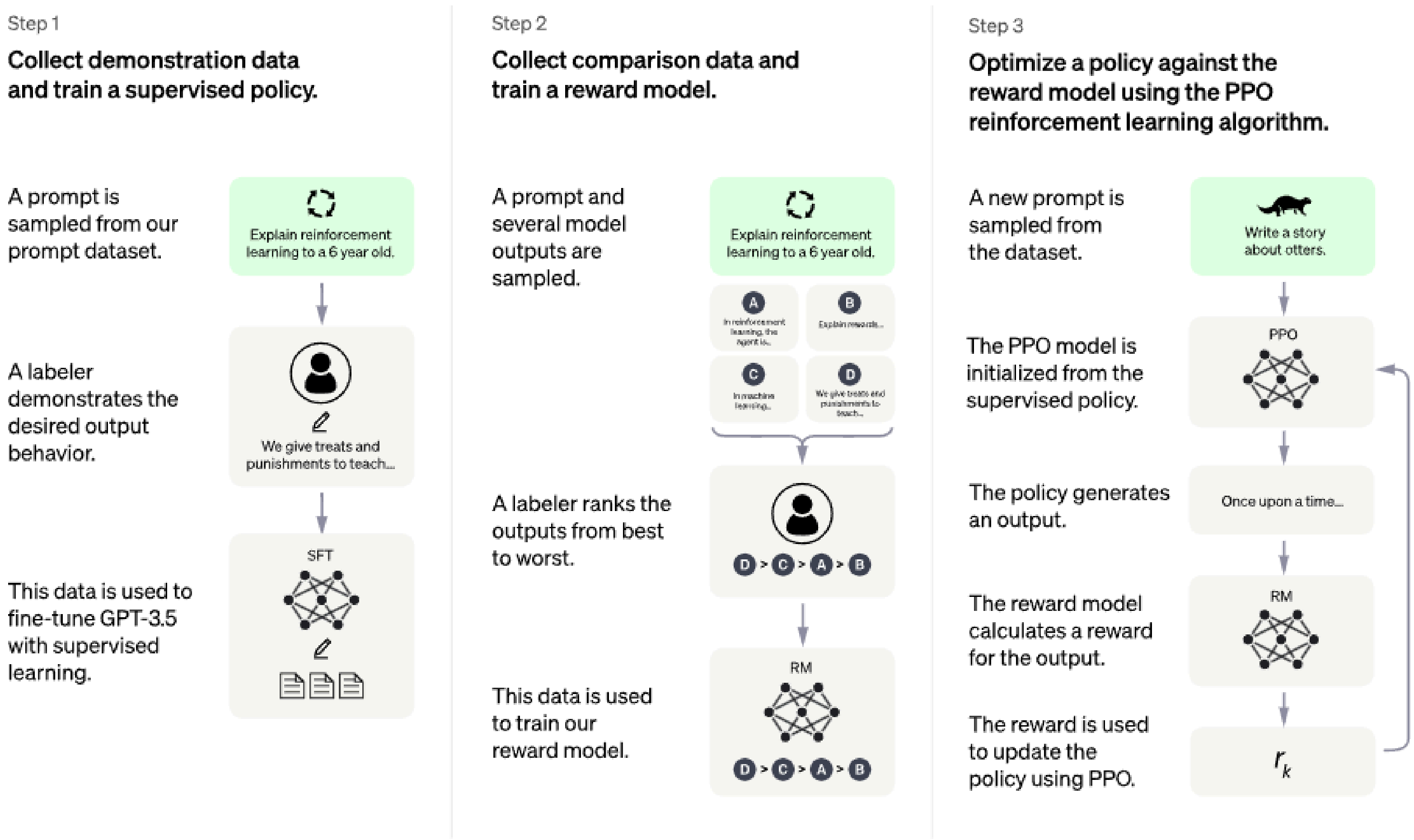
RL Application: Reinforcement Learning Using Human Feedback - Finetuning Chatgpt



Step 1

Collect demonstration data and train a supervised policy.

A prompt is sampled from our prompt dataset.

A labeler demonstrates the desired output behavior.

This data is used to fine-tune GPT-3.5 with supervised learning.

Step 2

Collect comparison data and train a reward model.

A prompt and several model outputs are sampled.

A labeler ranks the outputs from best to worst.

This data is used to train our reward model.

Step 3

Optimize a policy against the reward model using the PPO reinforcement learning algorithm.

A new prompt is sampled from the dataset.

The PPO model is initialized from the supervised policy.

The policy generates an output.

The reward model calculates a reward for the output.

The reward is used to update the policy using PPO.

Example copied verbatim from https://openai.com/blog/chatgpt.{style="font-size:30%"}

Summary of RL Algorithms

- Agent attempts to find **optimal policies** with highest returns via environment interaction
  - **Planning/Prediction** evaluates a given policy and **Learning/Control** finds the optimal policy
  - **Policy Iteration** for control involves value function estimation and policy improvement steps
- **Model-Free** learning does not require model of the environment (MDP)
  - **Monte Carlo (MC)** estimates the future returns by sampling returns via environment interaction
  - **Temporal Difference (TD)** estimates the future returns in a more online manner
  - **SARSA (On-policy)** and **Q-Learning (off-policy)** use MC/TD for model-free control
- **Model-Based** learning like **Dyna-Q** estimates the model of the environment (MDP)
- The **state-value**, **action-value functions** and **policies** can be approximated for large MDPs using neural networks or other parametric function approximators
- **Policy gradient** methods directly find optimal policies using gradient descent
- In practice, RL algorithms can be used in various applications like stock trading, self-driving cars and even systems like **ChatGPT**

References

- Based on the excellent RL book by Sutton and Barto:
  - http://incompleteideas.net/book/the-book-2nd.html
- Some content borrowed from David Silver's Lecture Notes:
  - https://www.davidsilver.uk/teaching/
- Additional help from Stanford CS234 course by Emma Brunskill:
  - https://web.stanford.edu/class/cs234/modules.html
- OpenAI Blogs:
  - https://openai.com/blog/chatgpt
  - https://spinningup.openai.com/en/latest/index.html