THE UNIVERSITY OF TEXAS AT AUSTIN
Department of Computer Science
College of Natural Sciences

Admixture of Poisson MRFs (APM): A Topic Model with Word Dependencies

David Inouye, Pradeep Ravikumar, Inderjit Dhillon

Motivation

- Previous topic models cannot model intuitive **dependencies between words**. (e.g. if the word "classification" occurs, "supervised" is more likely to occur.)
- Several **topic coherence metrics** that correlate with human judgment primarily *test* for word dependence [Minmo et al. 2011, Newman et al. 2010].

Contributions

1. Introduce **Admixture of Poisson MRFs (APM)** (a new topic model that considers *word dependencies*)
2. Formalize **admixtures** (a *generalization* of previous topic models)
3. Define a **novel conjugate prior** for a Poisson MRF
4. Develop **APM parameter estimation** method using an approximate MAP estimator
5. Show some preliminary qualitative and topic coherence results

Overview

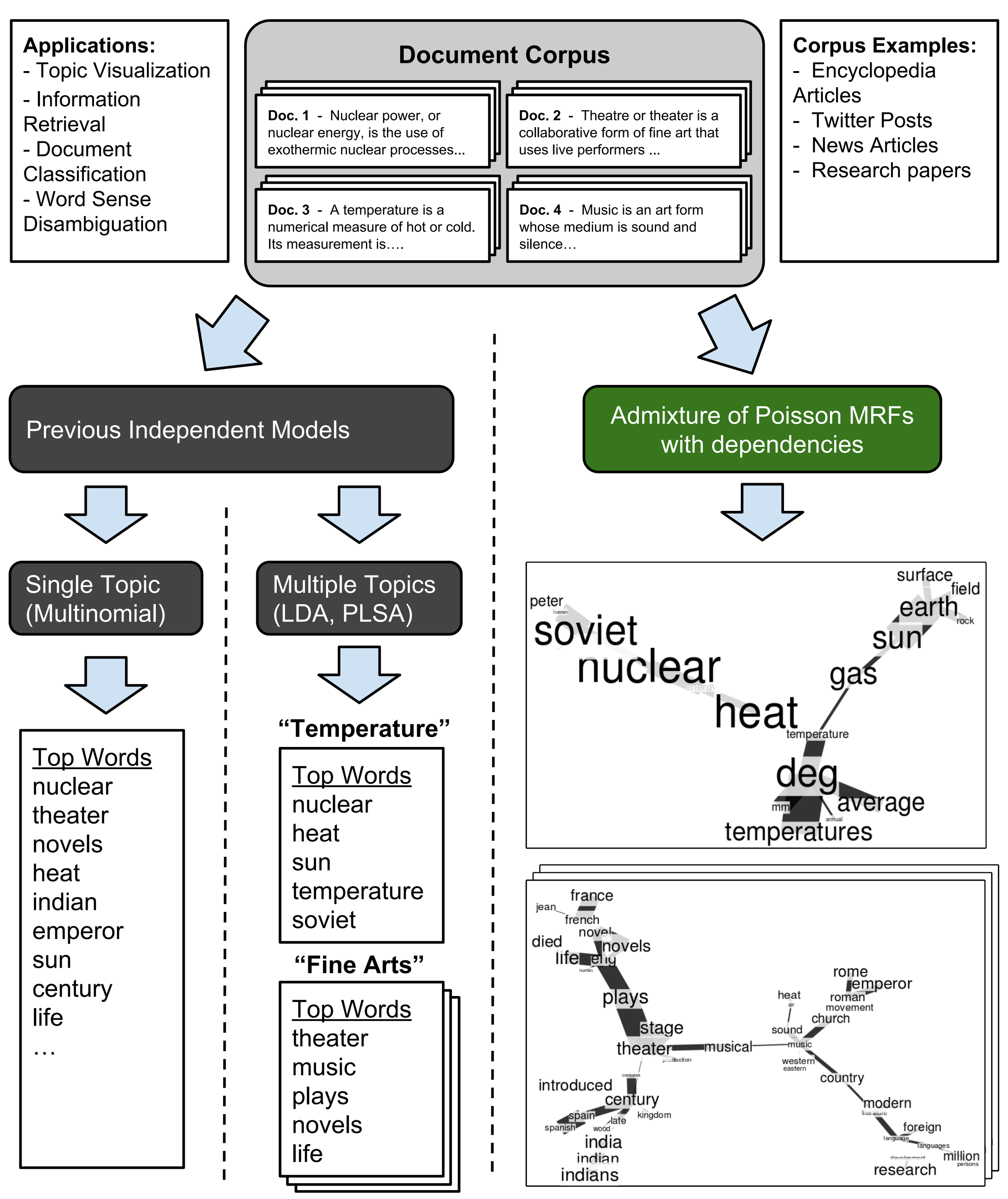


Figure: Previous topic models assume words are *independent* of each other and thus these previous models can only represent topics as a *list of words* ordered by frequency. However, our model, an admixture of Poisson MRFs, can model dependencies between words and hence can represent topics as a *graph over words*.

Generalized Admixtures

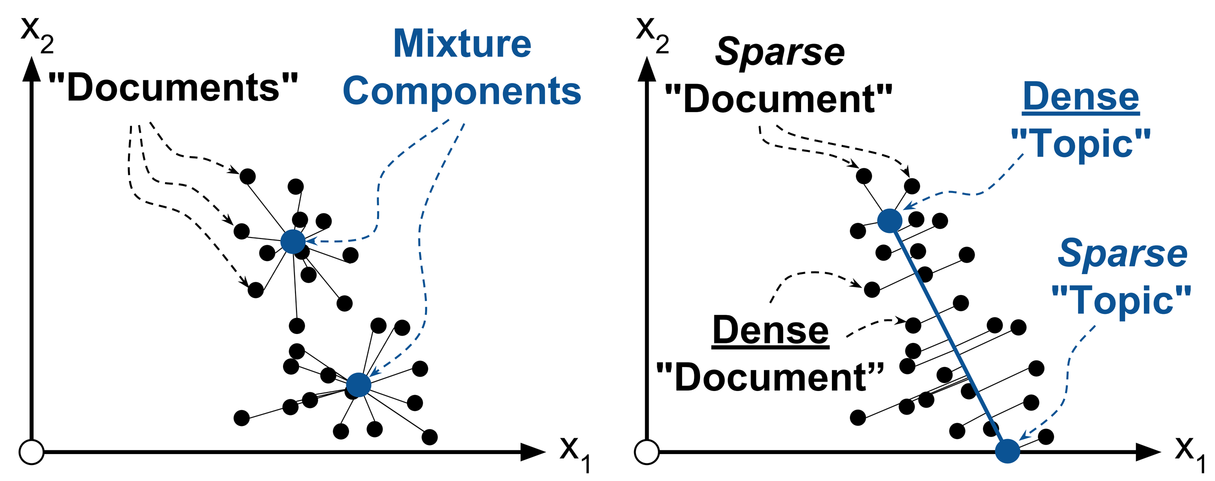


Figure: (Left) In *mixtures*, documents are drawn from exactly one component distribution. (Right) In *admixtures*, documents are drawn from a distribution whose parameters are a convex combination of component parameters.

The conditional distribution given the admixture weights and component distributions is merely the base distribution with parameters that are instance-specific mixtures of the component parameters:

$$\Pr_{\text{Admix.}}(\mathbf{x}\,|\,\mathbf{w},\Phi) = \Pr_{\text{Base}}\left(\mathbf{x}\,\middle|\,\bar\phi = \Psi^{-1}\Big[\sum_{j=1}^{k} w_j\Psi(\phi^j)\Big]\right)$$

Examples of admixtures/topic models:

- PLSA [Hofmann, 1999] - An admixture of Multinomials
- LDA [Blei et al. 2003] - An admixture of Multinomials with Dirichlet priors
- Spherical Admixture Model (SAM) [Reisinger et al., 2010] - An admixture of Von-Mises Fisher distributions

Background: Poisson MRF (Multivariate Poisson)

By assuming that the conditional distribution of a variable x_s given all other variables $\mathbf{x}_{\setminus s}$ is a univariate Poisson, a joint Poisson distribution can defined (Yang 2012, 2013):

$$\Pr_{\text{PMRF}}(\mathbf{x}\,|\,\boldsymbol\theta,\Theta) \propto \exp\left\{\boldsymbol\theta^T\mathbf{x} + \mathbf{x}^T\Theta\mathbf{x} - \sum_{s=1}^{p}\ln(x_s!)\right\},$$

where $\boldsymbol\theta \in \mathbb{R}^p$ and $\Theta \in \{\mathbb{R}^{p\times p} : \text{diag}(\Theta) = 0\}$.
Node conditionals (i.e. the distribution of one word given all other words) are 1-D Poissons:

$$\Pr(x_s\,|\,\mathbf{x}_{-s},\theta_s,\Theta_s) \propto \exp\{\underbrace{(\theta_s + \mathbf{x}^T\Theta_s)}_{\eta_s}x_s - \ln(x_s!)\}.$$

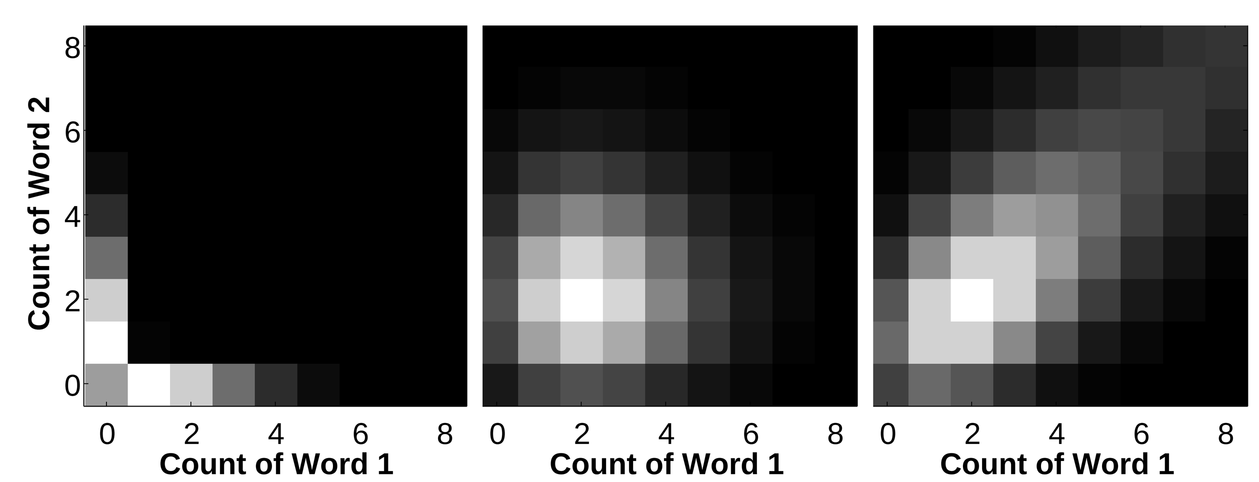


Figure: The densities of three 2D Poisson MRFs that show possible dependency structures between two words. Negative dependencies (left) suggest that two words *rarely* co-occur whereas positive dependencies (right) suggest that two words *often* co-occur.

Poisson MRFs in Context of LDA

LDA uses Multinomial distributions but if the parameter $N \sim \text{Poisson}(\tilde{x} = \sum_{s=1}^{p} x_s \,|\, \tilde\lambda = \sum_{s=1}^{p}\lambda)$, then the joint distribution is an independent Poisson model:[a]

$$\begin{aligned}
&\Pr_{\text{Poiss}}\left(\tilde x\,|\,\tilde\lambda\right)\Pr_{\text{Mult}}\left(\mathbf{x}\,|\,\theta = (\lambda_1,\cdots,\lambda_p)/\tilde\lambda, \mathrm{N} = \tilde x\right)\\
&= \frac{e^{-\tilde\lambda}}{\tilde x!}\tilde\lambda^{\tilde x}\frac{\tilde x!}{\prod_{s=1}^{p} x_s!}\prod_{s=1}^{p}\left(\frac{\lambda_s}{\tilde\lambda}\right)^{x_s}\\
&= \frac{\tilde x!\; e^{-\tilde\lambda}}{\tilde x!\prod_{s=1}^{p} x_s!}\prod_{s=1}^{p}\left(\frac{\tilde\lambda\lambda_s}{\tilde\lambda}\right)^{x_s}\\
&= \Pr_{\text{Ind. Poiss}}(\mathbf{x}\,|\,\lambda_1,\cdots,\lambda_p) = \prod_{s=1}^{p}\frac{e^{-\lambda_s}}{x_s!}\lambda_s^{x_s}
\end{aligned}$$

Therefore, the topic-word distributions of LDA can be viewed as special cases of Poisson MRFs.

Novel Conjugate Prior for PMRF

Form of a conjugate prior:

$$\Pr(\boldsymbol\theta,\Theta) \propto \exp\{\beta^T\boldsymbol\theta + \beta^T\Theta\beta - \gamma \mathrm{A}(\boldsymbol\theta,\Theta) - \lambda_{\boldsymbol\theta}\|\boldsymbol\theta\|_2^2 - \lambda\|\text{vec}(\Theta)\|_1\},$$

where $\mathrm{A}(\boldsymbol\theta,\Theta)$ is the log partition function of a PMRF.[b]

- $\lambda\|\text{vec}(\Theta)\|_1$ term encourages **sparsity** in Θ and is similar to adding a Laplace prior on Θ.
- $\boldsymbol\beta$ can be viewed as adding **pseudo-counts** to the observations similar to a Dirichlet prior for a Multinomial.

Admixture of Poisson MRFs

An Admixture of Poisson MRFs (APM) is an *admixture* with Poisson MRFs as the component distributions:

$$\Pr_{\text{APM}}(\mathbf{x},\mathbf{w},\boldsymbol\theta^{1\dots k},\Theta^{1\dots k}) =$$
$$\Pr_{\text{PMRF}}\left(\mathbf{x}\,\middle|\,\bar{\boldsymbol\theta} = \sum_{j=1}^{k} w_j\boldsymbol\theta^j, \bar\Theta = \sum_{j=1}^{k} w_j\Theta^j\right)\Pr_{\text{Dir}}(\mathbf{w})\prod_{j=1}^{k}\Pr(\boldsymbol\theta^j,\Theta^j)$$

Parameter Estimation

Parameter estimation is done by optimizing the approximate posterior (i.e. using pseudo log-likelihood) which has an ℓ_1 constraint, which enforces sparse parameters:

$$\mathop{\arg\min}_{\mathrm{W},\boldsymbol\theta^{1\dots k},\Theta^{1\dots k}} \underbrace{-\hat{\mathcal{L}}(\mathrm{W},\boldsymbol\theta^{1\dots k},\Theta^{1\dots k})}_{\text{differentiable}} + \underbrace{\delta_{\mathbb{W}}(\mathrm{W}) + \lambda\sum_{j=1}^{k}\|\Theta^j\|_1}_{\text{nonsmooth but convex}},$$

where $\hat{\mathcal{L}}$ is the pseudo log-likelihood and $\delta_{\mathbb{W}}(\mathrm{W})$ ensures that the weights are on the simplex. A proximal gradient method can be used to find a local minimum.

[a] Gopalan et al. (2013) recently introduced the connection between LDA and independent Poissons in the context of matrix factorization.
[b] $\lambda_{\boldsymbol\theta}\|\boldsymbol\theta\|_2^2$ and $\lambda\|\text{vec}(\Theta)\|_1$ needed for normalization of this prior distribution. In practice, $\lambda_{\boldsymbol\theta}$ can be set arbitrarily small and is thus ignored in subsequent discussion.

Qualitative Experiment

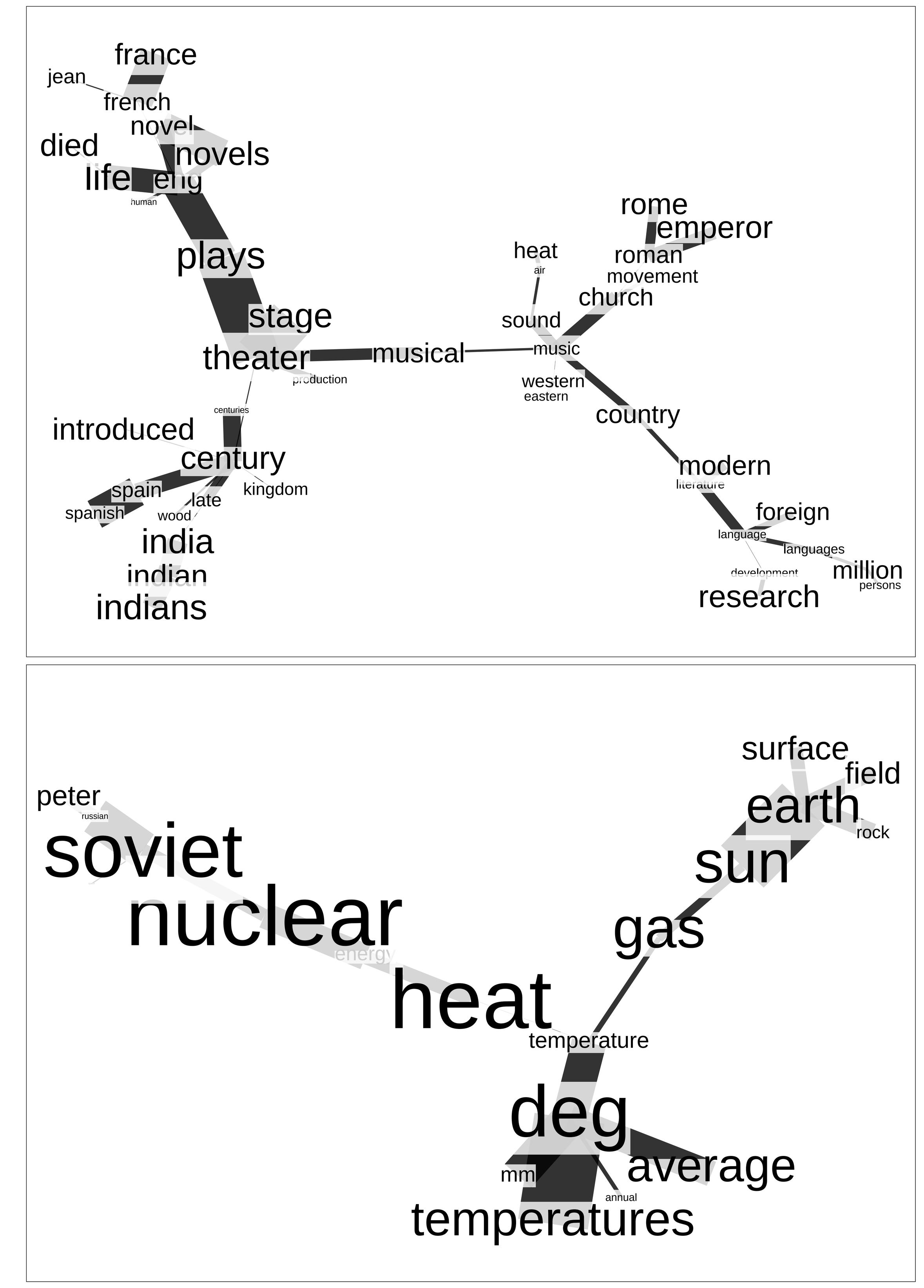


Figure: These APM topic visualizations ("Music and Fine Arts" and "Temperature") illustrate that PMRFs are much more intuitive than multinomials (as in LDA/PLSA), which can only be represented as a list of words. Word size signifies relative word frequency and edge width signifies the strength of word dependency (only positive dependencies shown).

Topic Coherence Experiments

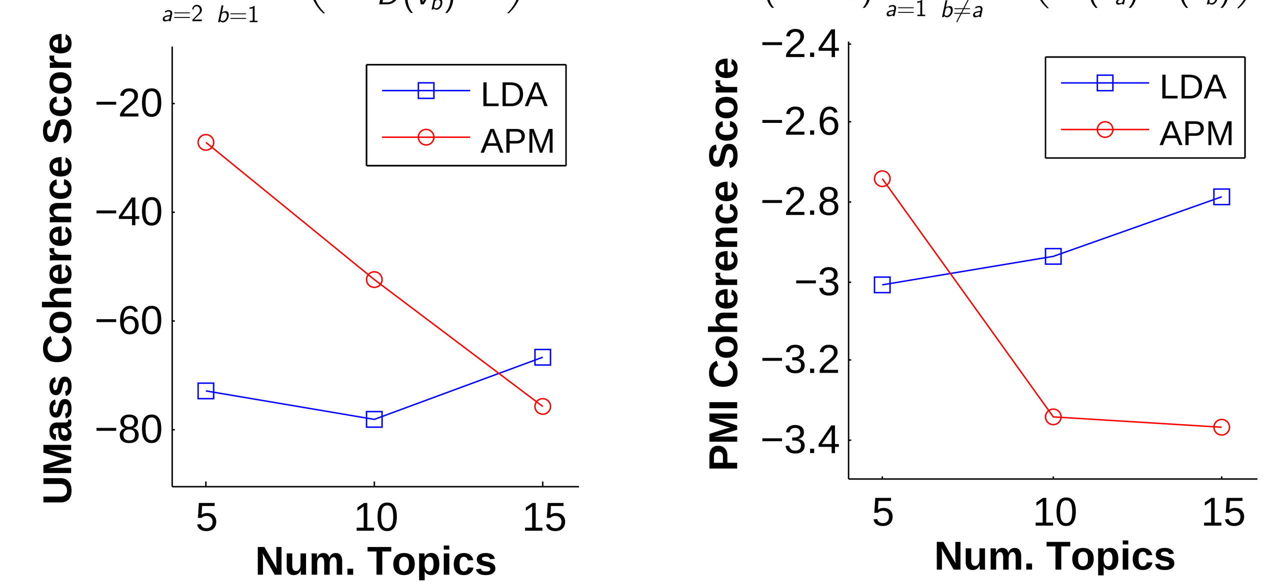
Dataset	# of Words	# of Documents
CMU 20 Newsgroup	200	18,846

- UMass Coherence Metric [Minmo et al. 2011]

$$\text{coh}_{\text{UMass}}(t) = \sum_{a=2}^{m}\sum_{b=1}^{a-1}\ln\left(\frac{D(v_a,v_b)+\epsilon}{D(v_b)}\right)$$

- Pointwise Mutual Info. [Newman et al. 2010]

$$\text{coh}_{\text{PMI}}(t) = \frac{1}{m(m-1)}\sum_{a=1}^{m}\sum_{b\neq a}\ln\left(\frac{\Pr(v_a,v_b)+\epsilon}{\Pr(v_a)\Pr(v_b)}\right)$$



For this preliminary experiment, APM seems to outperform LDA when the number of topics is small but is only comparable to LDA for a larger number of topics (Median scores shown).

http://www.cs.utexas.edu/~dinouye/
{dinouye,pradeepr,inderjit}@cs.utexas.edu